Algebra 2
The Polynomial Review

Name: KEY
Date: ______________ Period: ____

1.) What is the remainder when $p(x) = x^6 - 2x^3 + x - 1$ is divided by $(x + 1)$?

a.) -3
b.) -1
c.) 1
d.) 3

-1	1	0	0	-2	0	1	-1
		-1	1	-1	3	-3	2
	1	-1	1	-3	3	-2	1

2.) If $p(x) = x^3 - 2x^2 + 9x - 2$, which of the following statement(s) is/are true?

i. $x - 3$ is a factor of $p(x)$
ii. $x = 3$ is a root of $p(x)$
iii. $p(3) = 34$
iv. $p(-3) = 34$

3	1	-2	9	-2
		3	3	36
	1	1	12	34

a.) i only
b.) iii only
c.) i and ii only
d.) i and iii only
e.) i and iv only

3.) How many real roots must the following equation have?

$$x^4(x^2 - 4) + 9(x^2 - 4) = 0$$

$\underbrace{x^2 - 4 = 0}_{\text{2 REAL}}$ $\underbrace{x^4 + 9 = 0}_{\text{2 IMAGINARY}}$

a.) 1
b.) 2
c.) 4
d.) none

4.) Determine the quotient when $x^3 - 2x^2 - 9$ is divided by $(x - 3)$?

a.) $x^2 + 5x + 15$
b.) $x^2 + x - 6$
c.) $x^2 - 5x + 6$
d.) $x^2 + x + 3$

3	1	-2	0	-9
		3	3	9
	1	1	3	0

$x^2 + x + 3$

5.) What are the zeros of the polynomial function $f(x) = 2x^3 - 8x^2 + 6x$?

a.) $x = 0, 1, 3$
b.) $x = 1, 2, 3$
c.) $x = 0, -1, -3$
d.) $x = 0, 1, -4$

$2x(x^2 - 4x + 3) = 0$

$2x(x - 3)(x - 1) = 0$

$x = 0 \quad x = 3 \quad x = 1$

6.) Find the remainder when $f(x) = x^6 + 5x^5 - x^3 + x - 6$ is divided by $(x + 1)$.

a.) 0
b.) -10
c.) -1
d.) -12

$$\begin{array}{r|rrrrrrr} -1 & 1 & 5 & 0 & -1 & 0 & 1 & -6 \\ & & -1 & -4 & 4 & -3 & 3 & -4 \\ \hline & 1 & 4 & -4 & 3 & -3 & 4 & -10 \end{array}$$

7.) The polynomials $p(x) = x^4 + 5x^3 - 2x^2 - 24x$ has a zero at $x = 2$. Factor p completely.

a.) $p(x) = x(x + 2)(x + 3)(x + 4)$
b.) $p(x) = (x - 2)(x - 3)(x - 4)$
c.) $p(x) = x(x + 2)(x - 3)(x - 4)$
d.) $p(x) = x(x - 2)(x + 3)(x + 4)$

$$\begin{array}{r|rrrr} 2 & 1 & 5 & -2 & -24 \\ & & 2 & 14 & 24 \\ \hline & 1 & 7 & 12 & 0 \end{array}$$

x

$x^2 + 7x + 12 = 0$

$x(x-2)(x+3)(x+4)$

8.) For the given polynomials function, $f(x) = -5x^2(x - 8)(x + 2)^3$, find the zeros of the function and state the multiplicity of each.

0 8 -2
(2) (1) (3)

~~a.)~~ -2, *multiplicity* 1; 2, *multiplicity* 1; 8, *multiplicity* 1
b.) -2, *multiplicity* 3; 0, *multiplicity* 2; 8, *multiplicity* 1; 2, *multiplicity* 1
~~c.)~~ -2, *multiplicity* 1; 0, *multiplicity* 2; 8, *multiplicity* 1
d.) -2, *multiplicity* 3; 0, *multiplicity* 2; 8, *multiplicity* 1

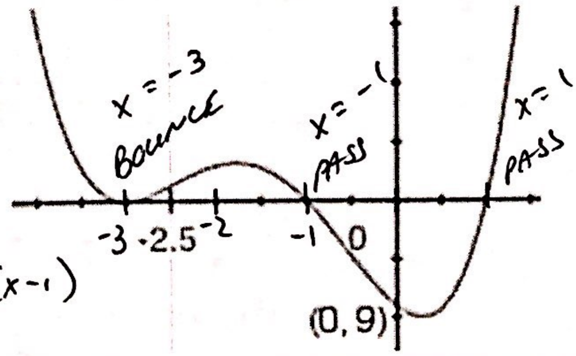
9.) For the given polynomials function, $f(x) = x^3 + 6x^2 - x - 6$, find the zeros of the function and state the multiplicity of each.

$x^2(x+6) - 1(x+6)$
$(x+6)(x^2-1)$
$(x+6)(x+1)(x-1)$

a.) -1, *multiplicity* 1; 1, *multiplicity* 1; 6, *multiplicity* 1
~~b.)~~ -6, *multiplicity* 2; 1, *multiplicity* 1
c.) -6, *multiplicity* 1; -1, *multiplicity* 1; 1, *multiplicity* 1
d.) -6, *multiplicity* 3; -1, *multiplicity* 1; 1, *multiplicity* 1

10.) The equation that would best model the following graph is:

a.) $f(x) = (x + 3)^2(x^2 - 1)$
b.) $f(x) = (x + 3)(x - 1)(x + 1)$
c.) $f(x) = x^4 + 6x^3 + 9x^2 - 6x - 9$
d.) $f(x) = -(x + 3)^2(x^2 - 1)$



$(x+3)(x+3)\ (x+1)(x-1)$
$(x+3)^2\ (x^2-1)$

11.) The function $f(x)$ has a zero of 2 with a multiplicity 3. We know...

~~a.)~~ Since 3 is an odd number, the graph touches but does not cross the x – axis.
b.) Since 3 is an odd number, the graph crosses the x – axis.
c.) Since 2 is an even number, the graph touches but does not cross the x – axis.
d.) Since 2 is an even number, the graph crosses the x – axis.

12.) The function $f(x)$ has a zero of 3 with a multiplicity 2. We know...

a.) Since the zero is 3, the graph crosses the y – axis at 3.
b.) Since the zeros is 3, the graph goes up to the right.
c.) Since the multiplicity is 2, the graph crosses the x – axis.
$\boxed{\text{d.)}}$ Since the multiplicity is 2, the graph touches but does not cross the x – axis.

13.) Using the polynomial, $f(x) = -2x^3 + 4x - 8$, explain how the degree and leading coefficient will affect the end behavior.

$\boxed{\text{a.)}}$ Because the degree is odd, the ends will point in opposite direction, and because the leading coefficient is negative the graph will point down on the right.

b.) Because the degree is odd, the ends will point in opposite direction, and because the leading coefficient is negative the graph will point up on the right.

c.) Because the degree is odd, the ends will point in the same direction, and because the leading coefficient is negative the graph will point down on the right.

d.) Because the degree is odd, the ends will point in the same direction, and because the leading coefficient is negative the graph will point up on the right.

14.) Determine the quartic function that is obtained from the parent function $y = x^4$ after the sequence of transformations.

a.) A vertical stretch by a factor of 3; a reflection across the x-axis; and a horizontal translation of 3 units right; and vertical translation of 2 units down

$$y = -3(x-3)^4 - 2$$

b.) A vertical shrink by a factor of $\frac{1}{3}$; a horizontal translation of 2 units left; and a vertical translation of 6 units up.

$$y = \frac{1}{3}(x+2)^4 + 6$$

15.) Divide using the synthetic division. Rewrite the polynomial function in factored form.

$(x^3 + 4x^2 + 14x + 20) \div (x + 2)$

$$\begin{array}{r|rrrr} -2 & 1 & 4 & 14 & 20 \\ & & -2 & -4 & -20 \\ \hline & 1 & 2 & 10 & 0 \end{array}$$

$x^2 + 2x + 10$

FACTORED FORM

$\rightarrow = (x+2)(x^2 + 2x + 10)$

16.) Find all zeros of the polynomials $f(x) = x^4 - 6x^3 + 25x^2 - 96x + 144$ given $x = 3$ is a zero of the function.

$$\begin{array}{c|ccccc} 3 & 1 & -6 & 25 & -96 & 144 \\ & & 3 & -9 & 48 & -144 \\ \hline & 1 & -3 & 16 & -48 & 0 \end{array}$$

$x^3 - 3x^2 + 16x - 48 = 0$

$x^2(x-3) + 16(x-3) = 0$

$(x-3)(x^2+16) = 0$

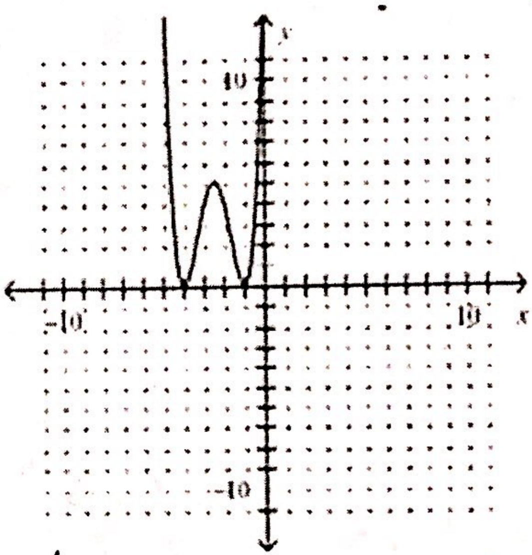
$x - 3 = 0 \quad x^2 + 16 = 0$

$x = 3 \quad x^2 = -16$

$x = \pm 4i$

Zeros: $3, 3, 4i, -4i$

17.) A complete graph of a polynomial function g is shown at the right.



a.) Is the degree of $g(x)$ even or odd? EVEN.
Explain: BOTH ENDS OF GRAPH GO IN SAME DIRECTION

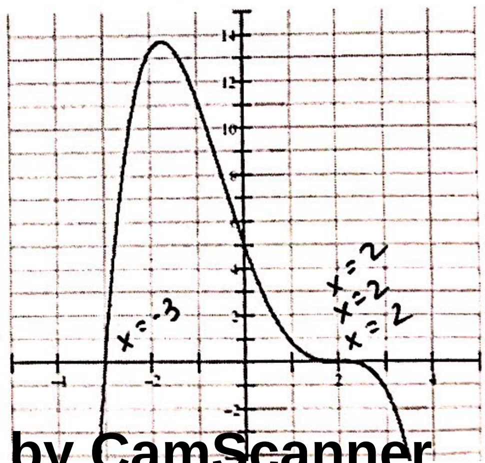
b.) Is the leading coefficient of $g(x)$ positive or negative? POSITIVE.
Explain: END BEHAVIOR OF GRAPH IS GOING TOWARD POSITIVE INFINITY

c.) What do the real zeros of $g(x)$ appear to be? $-4, -1$

d.) What is the smallest possible degree of $g(x)$? 4. Explain: GRAPH BOUNCES AT THE ZEROS INDICATING A REPEATING ANSWER

18.) Write the polynomial function of lowest degree in factored form for the following graph.

$f(x) = -(x+3)(x-2)^3$



19.) Using what you know about zeros, multiplicity, and end behavior draw a sketch of the graph of the following function:

$f(x) = 3(x-2)^3(x+4)^2$

x = 2 x = -4

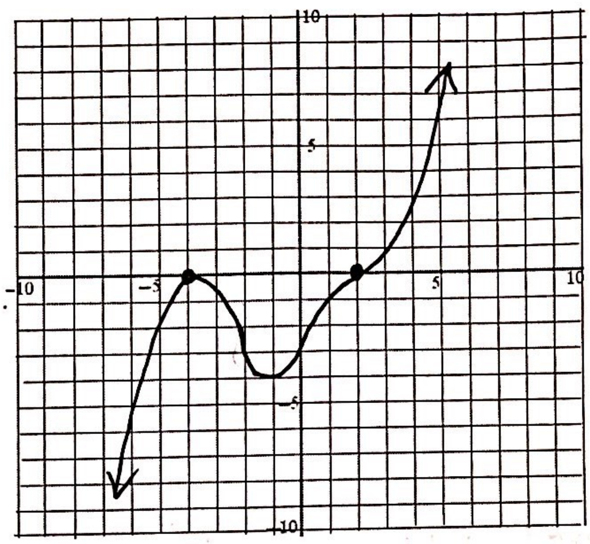
x = 2 x = -4

x = 2 BOUNCE

PASS

DEGREE : 5 (ODD)

L.C. : 3 (POSITIVE)



20.) Find all roots for $f(x) = x^3 + x^2 - 4x + 6$ given $(x+3)$ is a factor of the polynomial.

$$\begin{array}{r|rrrr} -3 & 1 & 1 & -4 & 6 \\ & & -3 & 6 & -6 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$x^2 - 2x + 2 = 0$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{4-8}}{2}$

$x = \dfrac{2 \pm \sqrt{-4}}{2}$

$x = \dfrac{2 \pm 2i}{2}$

$x = 1 \pm i$

ZEROS

-3

1 + i

1 - i

21.) Given the graph of the polynomial fill in the blanks below.

Degree: EVEN

Lead Coefficient: POSITIVE

End Behavior: $as\ x \to \infty, f(x) \to$ ∞

$as\ x \to -\infty, f(x) \to$ ∞

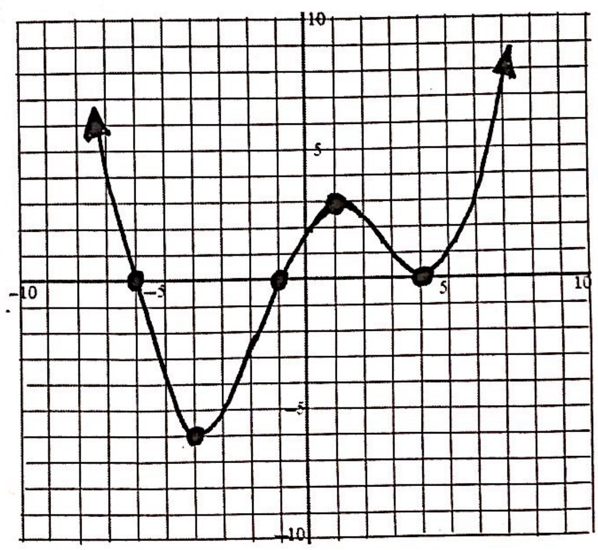
Zero(s): -6, -1, 4, 4

Relative Max: (1, 3)

Relative Min: (-4, -6) (4, 0)

Intervals Increasing: $(-4, 1) \cup (4, \infty)$

Intervals Decreasing: $(-\infty, -4) \cup (1, 4)$



22.) Given $f(x) = x^7 + 4x^6 - 2x^5 + x^4 - 2x^3 - 2x^2 - 3x + 5$ complete the table below with the possible combinations of real zeros and complex zeros.

Real Zeros	Complex Zeros	Total Zeros
7	0	7
5	2	7
3	4	7
1	6	7

23.) A jewelry box has a length that is 2 inches longer than the width and a height that is 1 inch smaller than the width. The volume of the box is 140 cubic inches. Write a polynomial function in standard form that models the above information? (Hint: $V = lwh$)

LENGTH: $x + 2$

WIDTH: x

HEIGHT: $x - 1$

$V = l \cdot w \cdot h$

$140 = x(x+2)(x-1)$

$140 = x(x^2 - x + 2x - 2)$

$140 = x(x^2 + x - 2)$

$140 = x^3 + x^2 - 2x$

$0 = x^3 + x^2 - 2x - 140$

24.) Factor the following expressions:

a.) $x^3 - 8$

$x\,x\,x \quad 2\,2\,2$

$(x-2)(x^2+2x+4)$

b.) $27x^3 - 125$

$3x\,3x\,3x \quad 5\,5\,5$

$(3x-5)(9x^2+15x+25)$

c.) $8x^3 + 1$

$2x\,2x\,2x \quad 1\,1\,1$

$(2x+1)(4x^2-2x+1)$

25.) Solve.

a.) $x^4 - 23x^2 = 50$

$x^4 - 23x^2 - 50 = 0$

$(x^4 - 25x^2) + (2x^2 - 50) = 0$

$x^2(x^2-25) + 2(x^2-25) = 0$

$(x^2-25)(x^2+2) = 0$

$(x+5)(x-5)(x^2+2) = 0$

$x+5 = 0 \quad x-5 = 0 \quad x^2+2 = 0$

$x = -5 \quad x = 5 \quad \sqrt{x^2} = \sqrt{-2}$

$x = \pm i\sqrt{2}$

b.) $2x^3 - 9x^2 = -9x$

$2x^3 - 9x^2 + 9x = 0$

$x(2x^2 - 9x + 9) = 0$

$x[2x^2 - 6x - 3x + 9] = 0$

$x[2x(x-3) - 3(x-3)] = 0$

$x(x-3)(2x-3) = 0$

$x = 0 \quad x-3 = 0 \quad 2x-3 = 0$

$x = 3 \quad x = 3/2$