

Verifying Trigonometric Identities

Review.

$$1. \quad \frac{x^5}{x^2} = x^{5-2}$$

$$= \boxed{x^3}$$

$$2. \quad \frac{x^7 y^4}{x^2} = x^{7-2} y^4$$

$$= \boxed{x^5 y^4}$$

$$3. \quad \frac{x^4 - 6}{x} = \frac{x^4}{x} - \frac{6}{x}$$

$$= \boxed{x^3 - \frac{6}{x}}$$

$$4. \quad (x-3)^2$$

$$= (x-3)(x-3)$$

$$= x^2 - 3x - 3x + 9$$

$$= \boxed{x^2 - 6x + 9}$$

$$5. \quad x^2 - 64$$

$$= \boxed{(x-8)(x+8)}$$

$$6. \quad 25x^4 - 49y^6$$

$$= \boxed{(5x^2 - 7y^3)(5x^2 + 7y^3)}$$

Given $\sin^2\theta + \cos^2\theta = 1$, generate the other 8 Pythagorean identities.

$$\blacksquare \sin^2\theta + \cos^2\theta = 1$$

$$\blacksquare \sec^2\theta - \tan^2\theta = 1$$

$$\blacksquare \csc^2\theta - \cot^2\theta = 1$$

$$\blacksquare \sin^2\theta = 1 - \cos^2\theta$$

$$\blacksquare \sec^2\theta = 1 + \tan^2\theta$$

$$\blacksquare \csc^2\theta = 1 + \cot^2\theta$$

$$\blacksquare \cos^2\theta = 1 - \sin^2\theta$$

$$\blacksquare \tan^2\theta = \sec^2\theta - 1$$

$$\blacksquare \cot^2\theta = \csc^2\theta - 1$$

Prove.

1. $\tan^2 \theta = \sec^2 \theta - 1$

▣ $\tan^2 \theta = \tan^2 \theta$

2. $\tan^2 \theta = \frac{1 - \cos^2 \theta}{\cos^2 \theta}$

▣ $\tan^2 \theta = \frac{1}{\cos^2 \theta} - \frac{\cos^2 \theta}{\cos^2 \theta}$

▣ $\tan^2 \theta = \sec^2 \theta - 1$

▣ $\tan^2 \theta = \tan^2 \theta$

3. $\tan \theta = \sin \theta \sec \theta$

▣ $\tan \theta = \sin \theta \cdot \frac{1}{\cos \theta}$

▣ $\tan \theta = \frac{\sin \theta}{\cos \theta}$

▣ $\tan \theta = \tan \theta$

4. $\frac{\cot^2 \theta}{\csc \theta} = \csc \theta - \sin \theta$

▣ $\frac{\csc^2 \theta - 1}{\csc \theta} = \csc \theta - \sin \theta$

▣ $\frac{\csc^2 \theta}{\csc \theta} - \frac{1}{\csc \theta} = \csc \theta - \sin \theta$

▣ $\csc \theta - \sin \theta = \csc \theta - \sin \theta$

5. $\frac{\cot \theta + 1}{\cot \theta} = 1 + \tan \theta$

▣ $\frac{\cot \theta}{\cot \theta} + \frac{1}{\cot \theta} = 1 + \tan \theta$

▣ $1 + \tan \theta = 1 + \tan \theta$

6. $(\sin \theta + \cos \theta)^2 = 2 \sin \theta \cos \theta + 1$

▣ $(\sin \theta + \cos \theta)(\sin \theta + \cos \theta) = 2 \sin \theta \cos \theta + 1$

▣ $\sin^2 \theta + \sin \theta \cos \theta + \cos \theta \sin \theta + \cos^2 \theta = 2 \sin \theta \cos \theta + 1$

▣ $2 \sin \theta \cos \theta + 1 = 2 \sin \theta \cos \theta + 1$

7. $(1 + \sin \theta)(1 - \sin \theta) = \cos^2 \theta$

▣ $1 - \sin \theta + \sin \theta - \sin^2 \theta = \cos^2 \theta$

▣ $1 - \sin^2 \theta = \cos^2 \theta$

▣ $\cos^2 \theta = \cos^2 \theta$

8. $\csc^4 \theta - \cot^4 \theta = \csc^2 \theta + \cot^2 \theta$

▣ $(\csc^2 \theta - \cot^2 \theta)(\csc^2 \theta + \cot^2 \theta) = \csc^2 \theta + \cot^2 \theta$

▣ $1(\csc^2 \theta + \cot^2 \theta) = \csc^2 \theta + \cot^2 \theta$

▣ $\csc^2 \theta + \cot^2 \theta = \csc^2 \theta + \cot^2 \theta$