Honors Algebra 2
Applications of Log-Exponential Functions

Name: KEY
Date: ________________ Period: __________

Population/Radioactivity:
$N = N_o e^{kt}$
N = Final number/amount
N_o = Initial number/amount
k = growth/decay constant
t = time

Money (Continuously compounded interest):
$A = Pe^{rt}$
A = Final amount
P = Principle (initial) amount
r = interest rate (decimal)
t = time

1.) The number of trees, N, of a given species per acre is approximated by the model, $N = 68(10^{-0.04x})$, where x ($5 \le x \le 40$) is the average diameter of the trees (in inches) measured 3 feet above the ground. Approximate the average diameter of a population of trees with a population density of 21 trees per acre.

- $21 = 68(10^{-.04x})$
- $\frac{21}{68} = 10^{-.04x}$
- $\log \frac{21}{68} = -.04x$
- $x = \frac{\log \frac{21}{68}}{-.04}$
- $\boxed{x = 12.76 \text{ IN}}$

2.) Ms. Watson wants to buy a \$40,000 car. She has saved \$27,000. Find the amount of time it will take for her \$27,000 to grow to \$40,000 at 4% compounded continuously.

- $A = Pe^{rt}$
- $40000 = 27000 e^{.04t}$
- $\frac{40}{27} = e^{.04t}$
- $\ln \frac{40}{27} = .04t$
- $t = \frac{\ln \frac{40}{27}}{.04}$
- $\boxed{t = 9.83}$

3.) How many years, to the nearest tenth of a year, will it take any amount of money to triple if it is invested at 3.25% compounded continuously?

- $A = Pe^{rt}$
- $3 = 1e^{.0325t}$
- $\ln 3 = .0325t$
- $t = \frac{\ln 3}{.0325}$
- $\boxed{t = 33.8 \text{ YEARS}}$

4.) The amount of time for a sample of Mathium to decay to half its original mass is 256 years. How long does it take for a sample whose initial mass is 500 grams to decay to a mass of 400 grams?

- $N = N_o e^{kt}$
- $400 = 500 e^{-0.00271t}$
- $\frac{4}{5} = e^{-0.00271t}$
- $\ln \frac{4}{5} = -0.00271t$
- $t = \frac{\ln \frac{4}{5}}{-0.00271} = \boxed{82.34 \text{ YEARS}}$

DECAY CONSTANT
- $N = N_o e^{kt}$
- $1 = 2e^{k256}$
- $\frac{1}{2} = e^{k256}$
- $\ln \frac{1}{2} = k \cdot 256$
- $k = \frac{\ln \frac{1}{2}}{256}$
- $k = -0.00271$

5.) Exactly one year ago, the population in the town of Historyville was 62,500. The town population is currently 61,400, due to an unknown disease. Assume the population change is modeled by a continuous exponential function. Find the population's decay constant, and estimate the amount of time required for the population to be 40,000 people.

* $61400 = 62500e^{K1}$
* $\frac{614}{625} = e^K$
* $\ln\frac{614}{625} = K$
* $K = -0.0178$

* $40{,}000 = 62500e^{-0.0178t}$
* $16/25 = e^{-0.0178t}$
* $\ln 16/25 = -0.0178t$
* $t = \frac{\ln 16/25}{-0.0178}$

* DID NOT USE CURRENT POPULATION

* $\boxed{t = 25.07 \text{ YEARS}}$

-1

$\boxed{t = 24.07 \text{ YEARS}}$

6.) The growth in the percentage of college freshmen who reported involvement in volunteer work during their last year of high school can be modeled by $f(t) = 74.61 + 3.84\ln t$, where t represents the number of years since 1990 and $f(t)$ represents the percentage of students. In what year did 81.6% of students complete volunteer work?

* $81.6 = 74.61 + 3.84\ln t$
* $6.99 = 3.84\ln t$
* $\frac{6.99}{3.84} = \ln t$
* $e^{\frac{6.99}{3.84}} = t$

* $t = 6.174$
* $1990 + 6 = \boxed{1996}$

7.) A person learning certain information through repetition tends to retain the information very well directly after learning. Retention decreases as time passes. Suppose the number of items a person can recall is modeled by $P(t) = 300(2.8)^{-0.5t}$, where t is the number days and P(t) is the number of items remembered. If a person starts out remembering 300 items, how long will it take for them to remember only half of the items?

* $150 = 300(2.8)^{-0.5t}$
* $1/2 = (2.8)^{-0.5t}$
* $\log_{2.8} 1/2 = -0.5t$
* $t = \frac{\log_{2.8} 1/2}{-0.5}$

* $\boxed{t = 1.3464 \text{ DAYS}}$

8.) Suppose you overhear someone in class saying, "I must reject any negative answers for x when I solve an equation involving logs." Is this correct? Why or why not?

9. There are 600 bacteria in a dish in a biology lab at 2 pm. If the number of bacteria triples every hour, how many bacteria are in the dish at 9 pm? Write the exponential model that represents this situation and then answer the question.

- $1800 = 600e^{k \cdot 1}$
- $3 = e^{k}$
- $\ln 3 = k$
- $k = 1.0986$

$\boxed{N = 600e^{1.0986t}}$

$N = 600e^{1.0986(7)}$

$\boxed{N = 1312087}$

10. You buy a new car for $28,400. The value of the car decreases by 22% each year.

a. Write an exponential decay model giving the car's value V (in dollars) after t years.

$A = 28400(1-.22)^{t}$

$A = 28400(.78)^{t}$

b. What is the value of the car (to the nearest$100) after 4 years?

- $A = 28400(.78)^{4}$
- $A = 10512.28$

c. In approximately how many years is the car worth $3000? Set up an equation and solve for your value.

- $3000 = 28400(.78)^{t}$
- $\frac{15}{142} = (.78)^{t}$
- $\log_{0.78} \frac{15}{142} = t$
- $t = \frac{\log \frac{15}{142}}{\log .78}$

$\boxed{t = 9.04 \quad \text{YEARS}}$

11. From 1990 to 2000, the population P of California can be modeled by the following equation, where t is the number of years since 1990.

$$P = 29,816,000(1.0128)^{t}$$

a. What was the population in 1990?

- $P = 29,816,000(1.0128)^{0}$
- $\boxed{P = 29,816,000}$

b. What is the growth factor and annual percent increase?

$\boxed{1.28\%}$

c. Estimate the population in 1999 (to the nearest thousand).

- $P = 29,816,000(1.0128)^{9}$
- $\boxed{P = 33,432,019}$

12. If a person invests \$1250 in an account that pays 8% interest compounded continuously, find the balance after 19 years.

- $A = Pe^{Rt}$
- $A = 1250e^{.08(19)}$
- $\boxed{A = \$5715.28}$

13. \$25,000 is deposited in an account paying 5.5% interest compounded continuously. When will the account be worth \$40,000?

- $A = Pe^{Rt}$
- $40000 = 25000e^{.055t}$
- $8/5 = e^{.055t}$
- $\ln 8/5 = .055t$
- $t = \dfrac{\ln 8/5}{.055}$
- $\boxed{t = 8.55}$

14. How much money must be deposited now in an account paying 2.5% simple annual interest to have a balance of \$8000 after 9 years?

- $8000 = P(1.025)^9$
- $\boxed{\$6405.83}$

15. The projected worth (in millions of dollars) of a large estate is modeled by the equation $V = 127(1.05)^x$. The variable x represents the number of years since 1989.

a) What is the projected annual percent of growth, and what should the estate be worth in 2010 (to the nearest million)?

- 5%

- $V = 127(1.05)^{21}$
- $\boxed{V = 353.82 \text{ MILLION}}$

b) In what year will the estate be worth \$250 million?

- $250 = 127(1.05)^x$
- $\dfrac{250}{127} = (1.05)^x$
- $\log_{(1.05)}\left(\dfrac{250}{127}\right) = x$
- $\boxed{x = 13.88}$

16. The population of Timbuktu is growing by a rate of 0.32% annually. If there were 8,212,000 residents of the country in 1995, estimate how many people (to the nearest thousand) will be living in the city in 2012.

$N = N_0 e^{kt}$

$N = 8{,}212{,}000\, e^{.0032(17)}$

$\boxed{N = 8{,}671{,}107}$

17. Write an exponential function to model the situation. Then estimate the value of the function after 7 years (to the nearest whole number). "A population of 1320 rodents decreases at an annual rate of 14%."

$\boxed{N = 1320\, e^{-.14t}}$

$N = 1320\, e^{-.14(7)}$

$\boxed{N \approx 495}$

Function ________________

After 7 years: ____________

18. Ima Loser had a total debt of $420,000 in 1990. Between 1990 and 2002, Ima was able to reduce her debt 13% each year. Approximate Ima's debt in 1998 to the nearest $1000.

$A = 420000(.87)^8$

$\boxed{A = 137848.90}$

19. A piece of equipment costs $79,000 new but depreciates 20% per year in each succeeding year. When will the equipment be worth $30,000?

$30000 = 79000(.8)^t$

$30/79 = (.8)^t$

$\log_{.8}(30/79) = t$

$\boxed{t = 4.34}$