

Simplify.

1.)  $(2 - 3i)(4 + i)$   
 $8 + 2i - 12i - 3i^2$   
 $8 - 10i + 3$   
 $11 - 10i$

2.)  $\frac{(5-4i)(8-2i)}{(8+2i)(8-2i)}$   
 $\frac{40 - 10i - 32i + 8i^2}{64 - 16i + 16i - 4i^2}$   
 $\frac{32 - 42i}{68}$   
 $\frac{16 - 21i}{34}$

3.)  $i^{55}$   
 $i \cdot i^{54}$   
 $i \cdot (i^2)^{27}$   
 $i \cdot (-1)^{27}$   
 $i \cdot (-1)$   
 $-i$

4.)  $2(7 - 2i) - 4i(3 - 2i)$   
 $14 - 4i - 12i + 8i^2$   
 $14 - 4i - 12i - 8$   
 $6 - 16i$

5.)  $\frac{4+5i}{3i} \cdot \frac{i}{i}$   
 $\frac{4i + 5i^2}{3i^2}$   
 $\frac{-5 + 4i}{-3}$

6.)  $(3 - 4i^{23})(2i + 2i^{31})$   
 $6i + 6i^{31} - 8i^{24} - 8i^{54}$   
 $6i + 6i \cdot i^{30} - 8(i^2)^{12} - 8(i^2)^{27}$   
 $6i + 6i(i^2)^{15} - 8(i^2)^{12} - 8(i^2)^{27}$   
 $6i + 6i(-1)^{15} - 8(-1)^{12} - 8(-1)^{27}$   
 $6i - 6i - 8(1) - 8(-1)$   
 $6i - 6i - 8 + 8$   
 $0$

Factor completely. Write "PRIME" if not factorable.

7.)  $54c^3 - 16$   
 $2(27c^3 - 8)$   
 $2[(3c)^3 - (2)^3]$   
 $2(3c - 2)(9c^2 + 6c + 4)$

8.)  $16x^4 - 81y^4$   
 $(4x^2 + 9y^2)(4x^2 - 9y^2)$   
 $(4x^2 + 9y^2)(2x + 3y)(2x - 3y)$

9.)  $3x^2 - 25x + 8$   
 $3x^2 - 24x - 1x + 8$   
 $(3x^2 - 24x) + (-1x + 8)$   
 $3x(x - 8) - 1(x - 8)$   
 $(x - 8)(3x - 1)$

Solve.

10.)  $10x^2 - 8x - 24 = 0$   ~~$\frac{-10 \quad -60}{-4 \quad 6}$~~   
 $2[5x^2 - 4x - 12] = 0$   
 $2[5x^2 - 10x + 6x - 12] = 0$   
 $2[(5x^2 - 10x) + (6x - 12)] = 0$   
 $2[5x(x - 2) + 6(x - 2)] = 0$   
 $2(x - 2)(5x + 6) = 0$   
 $x - 2 = 0 \quad 5x + 6 = 0$

11.)  $2x^3 - 12x^2 - 10x + 60 = 0$   
 $(2x^3 - 12x^2) + (-10x + 60) = 0$   
 $2x^2(x - 6) - 10(x - 6) = 0$   
 $(x - 6)(2x^2 - 10) = 0$   
 $x - 6 = 0 \quad 2x^2 - 10 = 0$   
 $x = 6 \quad 2x^2 = 10$   
 $x^2 = 5$   
 $x = \pm\sqrt{5}$

12.)  $9x^2 + 81 = 0$   
 $9x^2 = -81$   
 $x^2 = -9$   
 $x = \pm\sqrt{-9}$   
 $x = \pm 3i$

Solve each quadratic equation. You may only use each method once (Factoring, Square Root, and Quadratic Formula).

7.)  $3 - 4(x + 6)^2 = 35$

$$-4(x + 6)^2 = 32$$

$$(x + 6)^2 = -8$$

$$x + 6 = \pm\sqrt{-8}$$

$$x = -6 \pm \sqrt{-8}$$

$$x = -6 \pm 2i\sqrt{2}$$

8.)  $4x^2 - 6x - 21 = 3x^2 - 2x$

$$x^2 - 4x - 21 = 0$$

$$x^2 - 7x + 3x - 21 = 0$$

$$(x^2 - 7x) + (3x - 21) = 0$$

$$x(x - 7) + 3(x - 7) = 0$$

$$(x - 7)(x + 3) = 0$$

$$x - 7 = 0 \quad x + 3 = 0$$

$$x = 7 \quad x = -3$$

$$\begin{array}{r} -21 \\ \times 3 \\ \hline -7 \\ \times -4 \\ \hline \end{array}$$

9.)  $5x^2 - 4x + 1 = 2x^2 + 4x + 4$

$$3x^2 - 8x - 3 = 0$$

$$3x^2 - 9x + x - 3 = 0$$

$$(3x^2 - 9x) + (x - 3) = 0$$

$$3x(x - 3) + 1(x - 3) = 0$$

$$(x - 3)(3x + 1) = 0$$

$$x - 3 = 0 \quad 3x + 1 = 0$$

$$x = 3 \quad x = -1/3$$

10.)  $10x^2 - 2x + 7 = 8x^2 - 2$

$$2x^2 - 2x + 9 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(9)}}{2(2)}$$

$$x = \frac{2 \pm \sqrt{4 - 72}}{4}$$

$$x = \frac{2 \pm \sqrt{-68}}{4} \rightarrow x = \frac{2 \pm 2i\sqrt{17}}{4}$$

Graph. State all important information.

11.)  $y = 3(x + 3)^2 - 6$

Opens: UP

Axis of Symmetry:  $x = -3$

Vertex:  $(-3, -6)$

Max/Min: MIN

Domain:  $(-\infty, \infty)$

Range:  $[-6, \infty)$

Increasing:  $(-3, \infty)$

Decreasing:  $(-\infty, -3)$

End Behavior:

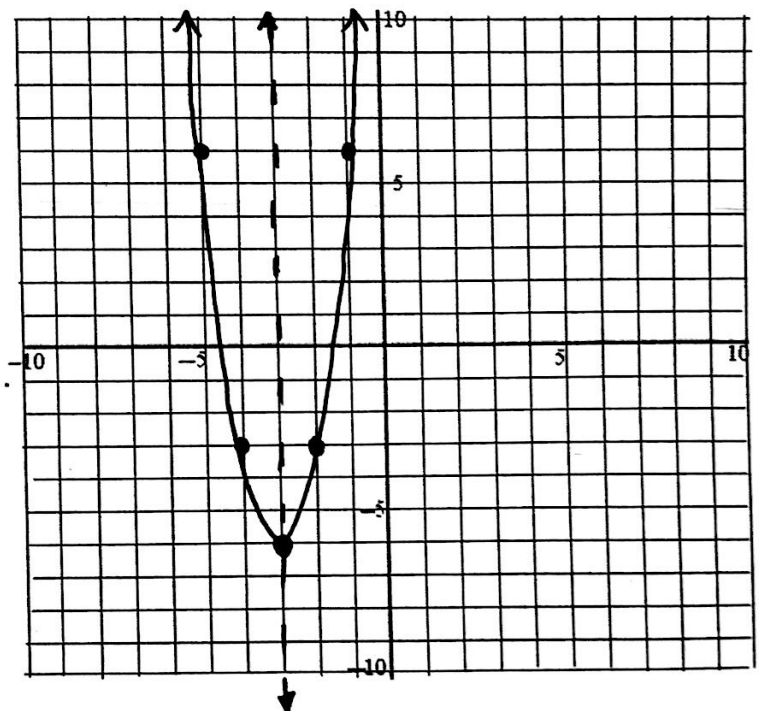
as  $x \rightarrow \infty$ ,  $f(x) \rightarrow \infty$

as  $x \rightarrow -\infty$ ,  $f(x) \rightarrow \infty$

Zero(s): \_\_\_\_\_

x	y
-2	-3
-1	6
0	21

$$x = \frac{1 + i\sqrt{17}}{2}$$



Solve the following system algebraically, then verify your solutions by finding them graphically.

$$12.) \quad y = -x^2 - 4x + 3$$

$$y = 2x - 3$$

Algebraically

$$2x - 3 = -x^2 - 4x + 3$$

$$2x + 3 = -x^2 - 2x + 6$$

$$0 = -x^2 - 6x + 6$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(-1)(6)}}{2(-1)}$$

$$x = \frac{6 \pm \sqrt{36 + 24}}{-2}$$

$$x = \frac{6 \pm \sqrt{60}}{-2}$$

$$x = -6.87 \quad x = .87$$

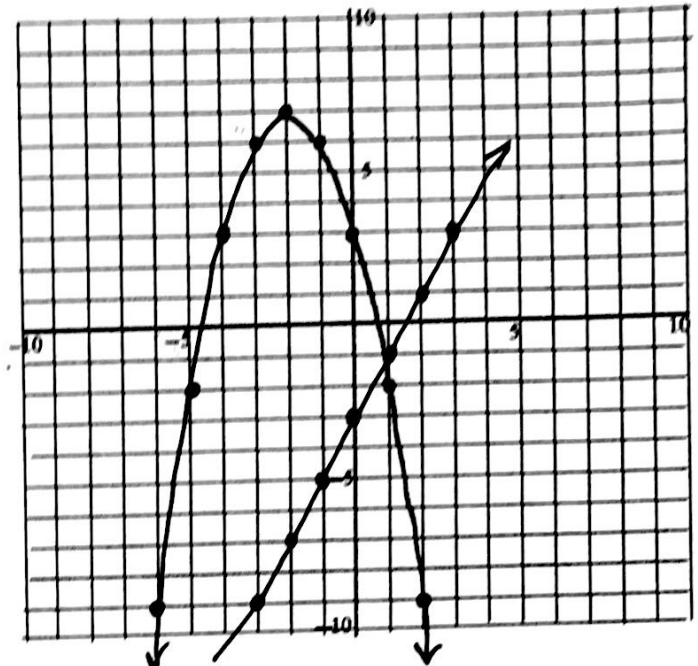
$$y = 2x - 3$$

$$y = 2(-6.87) - 3 \quad y = 2(.87) - 3$$

$$y = -16.74 \quad y = -1.26$$

Solution(s):  $(-6.87, -16.74) \quad (.87, -1.26)$

Graphically



Solution(s): \_\_\_\_\_

13.) A rocket launched by Mr. Fallinski carrying fireworks is launched from a hill 80 feet above Lake Carpentersville. At the rocket's maximum height it will explode and then fall back into the lake but not before displaying some of the most magnificent mathematical formulas ever seen. The rocket's height above the surface of the lake is given by  $h(t) = -16t^2 + 64t + 80$  where  $h(t)$  represent the height in feet in  $t$  seconds.

a.) What is the height of the rocket after 1.5 second?

$$h(1.5) = -16(1.5)^2 + 64(1.5) + 80$$

$$h(1.5) = 140 \text{ FT}$$

b.) When will the rocket reach its maximum height and what is the maximum height reached by the rocket?

$$x = \frac{-b}{2a} = \frac{-64}{2(-16)} = \frac{-64}{-32} = 2 \text{ seconds}$$

$$h(2) = -16(2)^2 + 64(2) + 80$$

$$h(2) = 144 \text{ FT}$$

c.) How long will it take for the rocket to hit 128 feet?

$$128 = -16t^2 + 64t + 80$$

$$0 = -16t^2 + 64t - 48$$

$$0 = -16(t^2 - 4t + 3)$$

$$0 = -16(t - 3)(t - 1)$$

$$t - 3 = 0 \quad t - 1 = 0$$

$$t = 3 \text{ sec} \quad t = 1 \text{ sec}$$

d.) After how many seconds after it is launched will the rocket hit the lake?

$$0 = -16t^2 + 64t + 80$$

$$0 = -16(t^2 - 4t - 5)$$

$$0 = -16(t - 5)(t + 1)$$

$$t - 5 = 0 \quad t + 1 = 0$$

$$t = 5 \quad t = -1$$

$$5 \text{ seconds}$$

- 14.) The profits of Mr. Trex's company can be represented by the equation  $y = -2(x - 12)^2 + 72$ , where  $y$  is the amount of profit in hundreds of thousands of dollars and  $x$  is the number of robotic dinosaur toys produced, measured in tens of thousands of units.

VERTEX : (12, 72)

- a.) How many units must be produced for Trex's business to earn the maximum profit?

$$12 \rightarrow 120,000 \text{ TOYS}$$

- b.) What is the maximum profit?

$$72 \rightarrow \$7,200,000$$

- c.) At what production level (number of units) will profit be zero?

$$0 = -2(x-12)^2 + 72$$

$$-72 = -2(x-12)^2$$

$$36 = (x-12)^2$$

$$\pm 6 = x - 12$$

$$x = 12 \pm 6$$

$$x = 12 + 6$$

$$x = 18$$

$$180,000 \text{ TOYS}$$

$$x = 12 - 6$$

$$x = 6$$

$$60,000 \text{ TOYS}$$

- d.) What is the average profit per unit produced and sold when the company is operating at peak profitability?

$$\frac{7200000}{120000} = \$60 \text{ PER UNIT}$$

- 15.) If  $f(x) = -3x^2 + 4x - 2$ , find  $f(m-2)$

$$\begin{aligned} f(m-2) &= -3(m-2)^2 + 4(m-2) - 2 \\ &= -3(m^2 - 4m + 4) + 4(m-2) - 2 \\ &= -3m^2 + 12m - 12 + 4m - 8 - 2 \\ &= -3m^2 + 16m - 22 \end{aligned}$$