Exponential Graph Discovery
H-Algebra 2

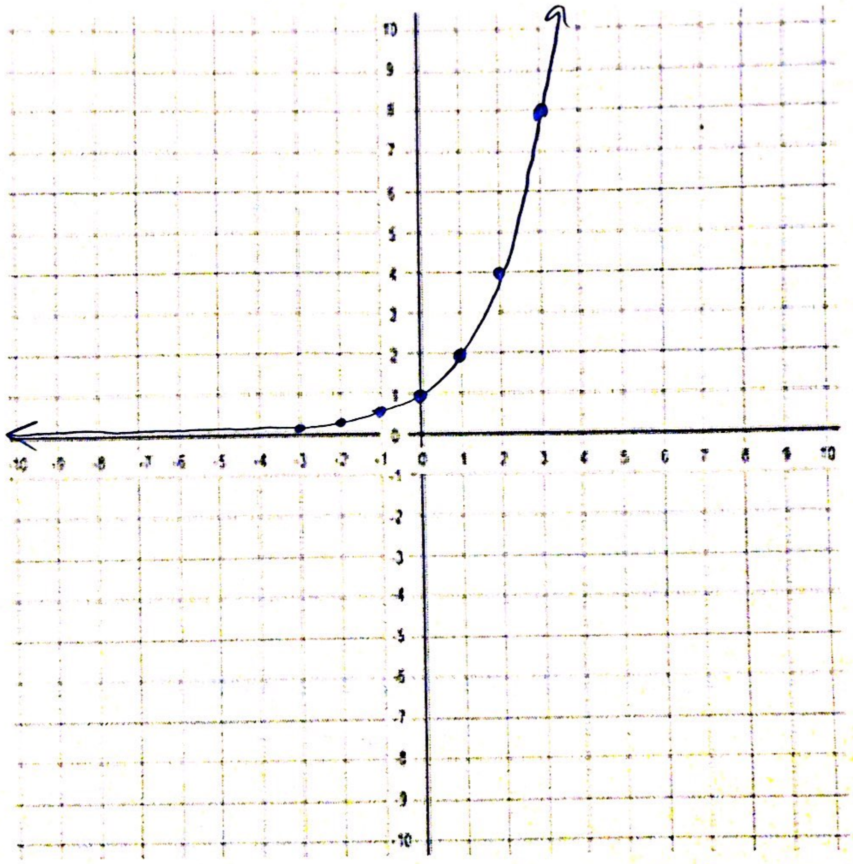
Ponce de Key
Date ______ Period ______

Fill in the tables WITH RATIONAL VALUES and use them to create a graph. Since we are graphing functions in function notation – f(x), etc – they are continuous functions. (What does that mean?) Use your graphs to answer the questions. You will be asked questions about the relationships among these graphs, so be sure to note *why* things change, not just how.

Connect the dots!

1. $f(x) = 2^x$

x	f(x) as fraction	f(x) as decimal approximation
-10	1/1024	0.000977
-9	1/512	0.00195
-8	1/256	0.00391
-7	1/128	0.00781
-6	1/64	0.0156
-5	1/32	0.03125
-4	1/16	0.0625
-3	1/8	0.125
-2	1/4	0.25
-1	1/2	0.5
0	1	1
1	2	2
2	4	4
3	8	8
4	16	16
5	32	32
6	64	64
7	128	128
8	256	256
9	512	512
10	1024	1024



Will this graph ever touch the x-axis? Why or why not?

No, it will not. According to the middle column, the fraction for f(x) will always have a non-zero numerator, so the fraction will not equal zero. Also, recall that $b^{-n} = \frac{1}{b^n}$

If not, is there an asymptote? What would its equation be?

There appears to be an asymptote at $y=0$.

State the DOMAIN of the function f(x) = 2^x in interval notation: $(-\infty, \infty)$

State the RANGE of the function f(x) = 2^x in interval notation: $(0, \infty)$

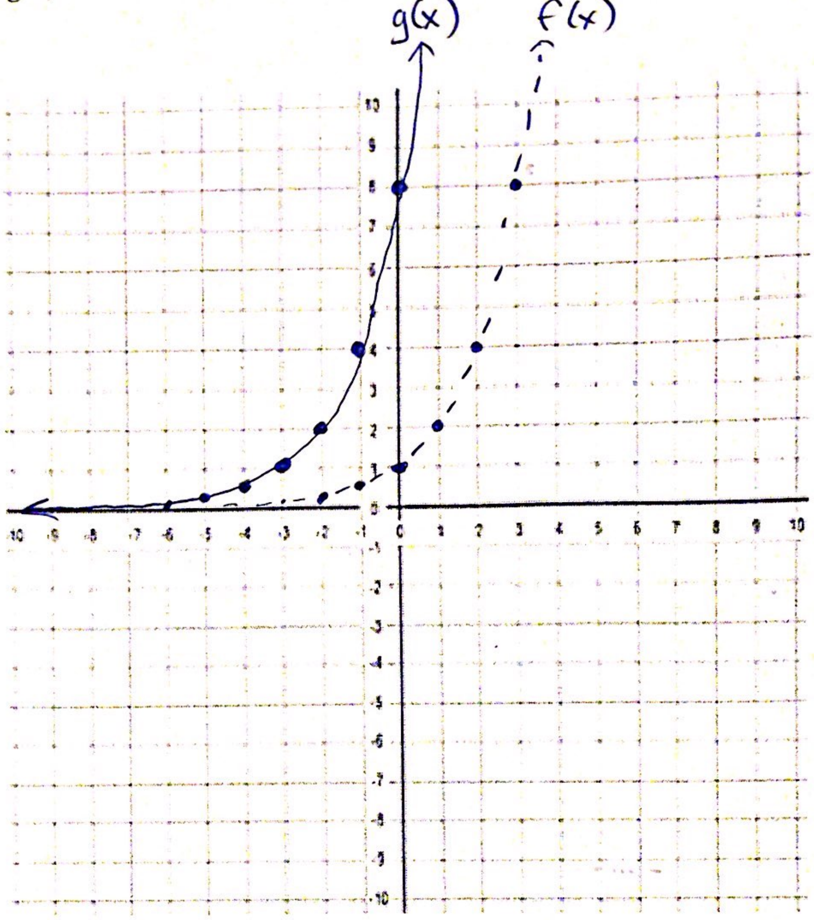
State the end behavior of the function f(x) = 2^x: as $x \to -\infty$, $f(x) \to$ 0

as $x \to \infty$, $f(x) \to$ ∞

Graph f(x) from the previous page with a light, dashed curve, then plot g(x) below with a solid curve.

2. $g(x) = 2^{x+3}$

x	g(x) as fraction	g(x) as decimal approximation
-10	1/128	0.00781
-9	1/64	0.0156
-8	1/32	0.03125
-7	1/16	0.0625
-6	1/8	0.125
-5	1/4	0.25
-4	1/2	0.5
-3	1	1
-2	2	2
-1	4	4
0	8	8
1	16	16
2	32	32
3	64	64
4	128	128
5	256	256
6	512	512
7	1024	1024
8	2048	2048
9	4096	4096
10	8192	8192



What are the similarities between f(x) and g(x)? Be specific.

They have the same basic shape. They both have asymptotes at y=0. They have the same domain (-∞, ∞) and range (0,∞). The y-values in the tables are nearly identical, but they repeat over different x-values.

What is different? What *transformation* occurred?

The location of the shape changed.

$g(x) = 2^{x+3}$ is the same as $f(x) = 2^x$ moved 3 spaces LEFT.

Equation of the asymptote: $y = 0$

State the DOMAIN of the function $g(x) = 2^{x+3}$ in interval notation: $(-\infty, \infty)$

State the RANGE of the function $g(x) = 2^{x+3}$ in interval notation: $(0, \infty)$

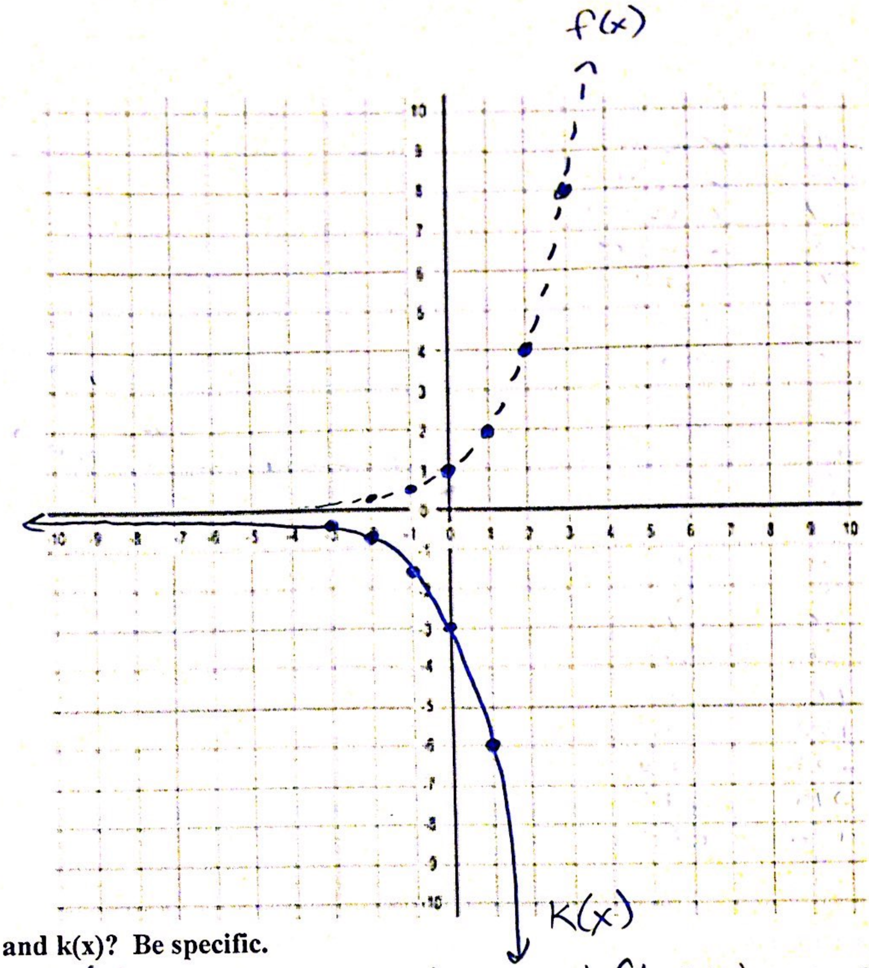
State the end behavior of the function $g(x) = 2^{x+3}$: as $x \to -\infty$, $g(x) \to$ 0

as $x \to \infty$, $g(x) \to$ ∞

Graph f(x) from the first page with a light, dashed curve, then plot k(x) below with a solid curve.

3. $k(x) = -3(2)^x$

x	k(x) as fraction	k(x) as decimal approximation
-10	-3/1024	-0.00293
-9	-3/512	-0.00586
-8	-3/256	-0.0117
-7	-3/128	-0.0234
-6	-3/64	-0.0469
-5	-3/32	-0.0938
-4	-3/16	-0.1875
-3	-3/8	-0.375
-2	-3/4	-0.75
-1	-3/2	-1.5
0	-3	-3
1	-6	-6
2	-12	-12
3	-24	-24
4	-48	-48
5	-96	-96
6	-192	-192
7	-384	-384
8	-768	-768
9	-1536	-1536
10	-3072	-3072



What are the similarities between f(x) and k(x)? Be specific.

The basic shapes are the same (Approaching asymptote on left, racing away on R)

The asymptotes are the same (y=0)

The domains (-∞, ∞) are the same.

What is different? What *transformation* occurred?

The points have been pulled further away from the x-axis (stretched) and flipped (reflected) to the other side.

Equation of the asymptote: y=0

State the DOMAIN of the function $k(x) = 3(2)^x$ in interval notation: (-∞, ∞)

State the RANGE of the function $k(x) = 3(2)^x$, in interval notation: (-∞, 0)

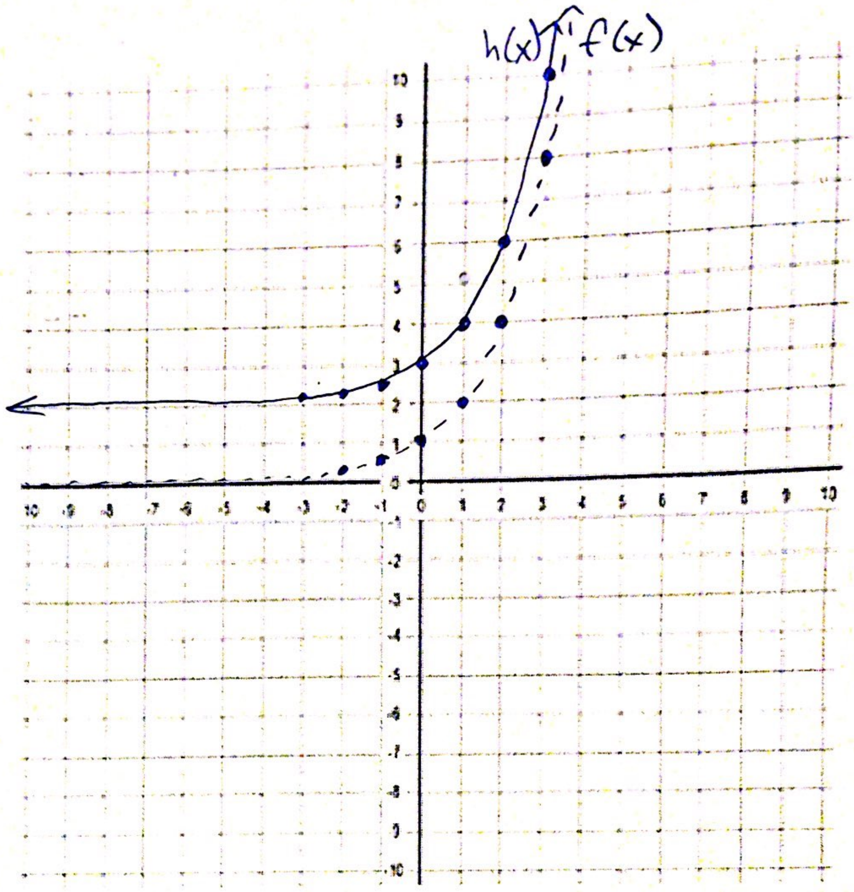
State the end behavior of the function $k(x) = 3(2)^x$: as $x \to -\infty$, $k(x) \to$ 0

as $x \to \infty$, $k(x) \to$ -∞

Graph f(x) from the first page with a light, dashed curve, then plot h(x) below with a solid curve.

4. $h(x) = 2^x + 2$

x	h(x) as fraction	h(x) as decimal approximation
-10	2049/1024	2.000977
-9	1025/512	2.00195
-8	513/256	2.00391
-7	257/128	2.00781
-6	129/64	2.0156
-5	65/32	2.03125
-4	33/16	2.0625
-3	17/8	2.125
-2	9/4	2.25
-1	5/2	2.5
0	3	3
1	4	4
2	6	6
3	10	10
4	18	18
5	34	34
6	66	66
7	130	130
8	258	258
9	514	514
10	1026	1026



What are the similarities between f(x) and h(x)? Be specific.

They have the same basic shape

They have the same domains $(-\infty, \infty)$

What is different? What *transformation* occurred?

It appears that $h(x) = 2^x + 2$ is the same graph as $f(x) = 2^x$ moved UP two units.

Equation of the asymptote: $y = 2$

State the DOMAIN of the function $h(x) = 2^x + 2$ in interval notation: $(-\infty, \infty)$

State the RANGE of the function $h(x) = 2^x + 2$ in interval notation: $(2, \infty)$

State the end behavior of the function $h(x) = 2^x + 2$: as $x \to -\infty$, $h(x) \to$ 2

as $x \to \infty$, $h(x) \to$ ∞

Predict the transformations that that will be applied to the *parent function*. List them in the order that they will occur (think PEMDAS!!!)

5. $y = 4(2)^{x+5} - 3$ **Transformations:**

Left 5
Vertical Stretch of ×4
Down 3

6. $y = -(2)^{x-4} - 6$ **Transformations:**

Right 4
Flip (reflect) over x-axis (no stretch)
Down 6

7. $y = \frac{1}{2}(2)^{x+1} + 4$ **Transformations:**

Left 1
Vertical Stretch of $\times\frac{1}{2}$ (shrink?)
Up 4

8. $y = 2^{x+3} - 7$ **Transformations:**

Left 3

Down 7

Plot points and graph f(x) = 2^x lightly with a dashed curve. Then use the transformations to plot $t(x) = 3(2)^{x-1} - 8$

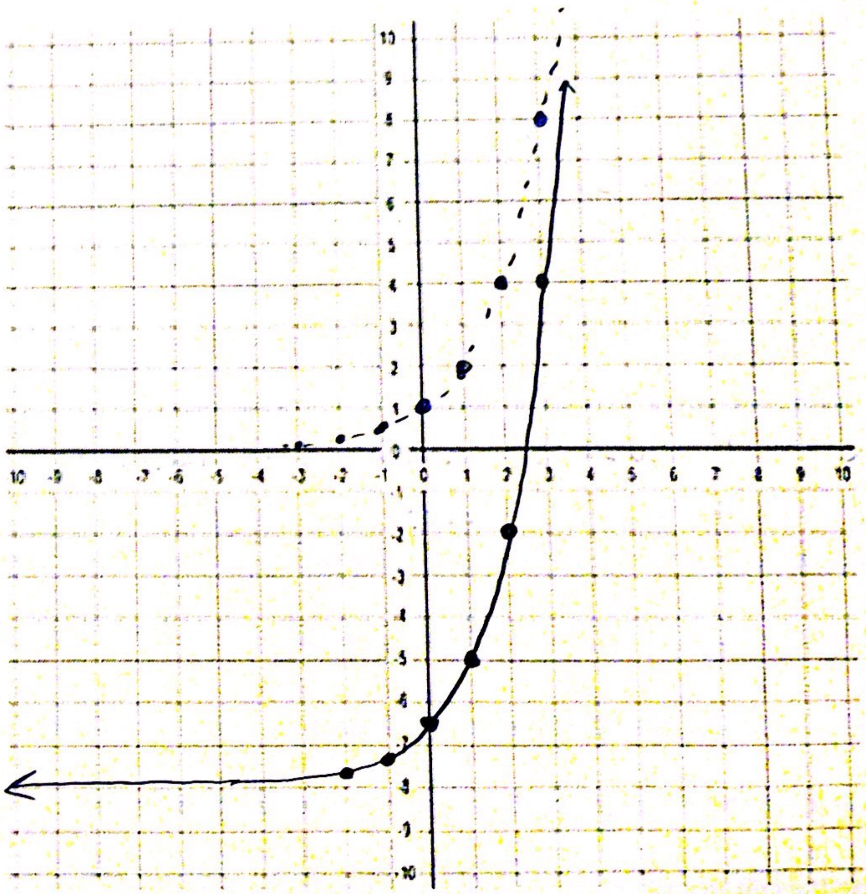
$t(x) = 3(2)^{x-1} - 8$

Transformations:

Right 1

Vertical Stretch of ×3

Down 8



9. Simplify/evaluate without a calculator.

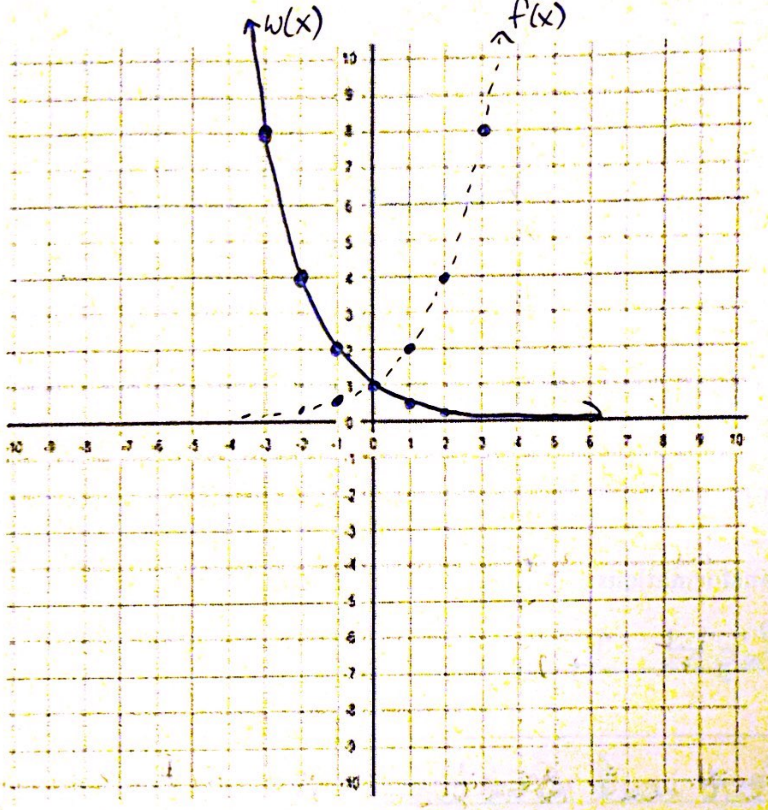
a) $(2)^1 = 2$ b) $(2)^{-1} = \frac{1}{2^1} = \frac{1}{2}$ c) $\left(\frac{1}{2}\right)^1 = \frac{1}{2}$ d) $\left(\frac{1}{2}\right)^{-1} = \left(\frac{2}{1}\right)^1 = 2$

e) $(2)^3 = 8$ f) $(2)^{-3} = \frac{1}{2^3} = \frac{1}{8}$ g) $\left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \frac{1}{8}$ h) $\left(\frac{1}{2}\right)^{-3} = \frac{2^3}{1^3} = \frac{8}{1} = 8$

i) $a^4 = a^4$ j) $a^{-4} = \frac{1}{a^4}$ k) $\left(\frac{a}{b}\right)^4 = \frac{a^4}{b^4}$ l) $\left(\frac{a}{b}\right)^{-4} = \frac{b^4}{a^4}$

10. $w(x) = \left(\frac{1}{2}\right)^x$

x	w(x) as fraction	w(x) as decimal approximation
-6	64	64
-5	32	32
-4	16	16
-3	8	8
-2	4	4
-1	2	2
0	1	1
1	1/2	0.5
2	1/4	0.25
3	1/8	0.125
4	1/16	0.0625
5	1/32	.03125
6	1/64	.0156



Will this graph ever touch the x-axis? Why or why not?

It will not touch the x-axis. The numerator of the fraction representing w(x) will never be zero, so w(x) will never be zero.

If not, is there an asymptote? What would its equation be?

The asymptote is $y=0$

State the DOMAIN of the function f(x) = (1/2)^x in interval notation: $(-\infty, \infty)$

State the RANGE of the function f(x) = (1/2)^x in interval notation: $(0, \infty)$

State the end behavior of the function f(x) = (1/2)^x: as $x \to -\infty$, $f(x) \to$ ∞

as $x \to \infty$, $f(x) \to$ 0