Algebra 2

Final Exam Review – Semester 1 – 3415

Name: KEY

Date: ______________ Period: ______

1.) A rock is thrown through the air by a large catapult. The path of the boulder can be modeled by the quadratic equation $s(t) = -16t^2 + 96t + 128$, where $s(t)$ represents the rock's height (in feet) after t seconds.

a.) When does the rock reach its maximum height?

$x = \frac{-96}{2(-16)} = \frac{-96}{-32} = 3$

$\boxed{\text{3 SECONDS}}$

b.) What is the maximum height?

$s(3) = -16(3)^2 + 96(3) + 128$

$= 272$

$\boxed{\text{272 FEET}}$

c.) Write the equation in vertex form.

$\boxed{s(t) = -16(t-3)^2 + 272}$

d.) When will the rock land?

$0 = -16t^2 + 96t + 128$

$0 = -16(t^2 - 6t - 8)$

$\boxed{\text{7.123 SEC}}$

$t^2 - 6t - 8 = 0$

$t^2 - 6t + \underline{9} = 8 + \underline{9}$

$(t-3)^2 = 17$

$t - 3 = \pm\sqrt{17}$

$t = 3 \pm \sqrt{17}$

$t = 3 + \sqrt{17}$ $\quad t = 3 - \sqrt{17}$

$t = 7.123$ $\quad t = -1.123$

e.) When will the rock be at an altitude of 80 feet?

$80 = -16t^2 + 96t + 128$

$0 = -16t^2 + 96t + 48$

$0 = -16(t^2 - 6t - 3)$

$t^2 - 6t - 3 = 0$

$t^2 - 6t + \underline{9} = 3 + \underline{9}$

$(t-3)^2 = 12$

$t - 3 = \pm\sqrt{12}$

$t = 3 \pm \sqrt{12}$

$t = 3 + \sqrt{12}$ $\quad t = 3 - \sqrt{12}$

$t = 6.46$ $\quad t = -0.46$

$\boxed{\text{6.46 SEC}}$

Define variables, write a system of equations and then solve.

2.) An instructor wants to write a quiz with 9 questions where each question is worth 3, 4, or 5 points based on difficulty. He wants the number of 3-point questions to be twice as many the number of 5-point questions, and he wants the quiz to be worth a total of 35 points. How many 3, 4, and 5 point questions could there be?

x = 3 PT QUESTION

y = 4 PT QUESTION

z = 5 PT QUESTION

$x + y + z = 9$

$x = 2z$

$3x + 4y + 5z = 35$

$2z + y + z = 9$

$3z + y = 9$

$3(2z) + 4y + 5z = 35$

$11z + 4y = 35$

$-12z - 4y = -36$

$11z + 4y = 35$

$-1z = -1$

$z = 1$

$3(1) + y = 9$

$y = 6$

$x + 6 + 1 = 9$

$x = 2$

$\boxed{(2, 6, 1)}$

Perform the indicated operation.

3.) $(5-2i)(3i+4)$

$15i+20-6i^2-8i$

$15i+20+6-8i$

$\boxed{26+7i}$

4.) $(4i+3)^2$

$(4i+3)(4i+3)$

$16i^2+12i+12i+9$

$-16+12i+12i+9$

$\boxed{-7+24i}$

5.) $\frac{(4-i)}{(2i+3)} \cdot \frac{(2i-3)}{(2i-3)}$

$\frac{8i-12-2i^2+3i}{4i^2-6i+6i-9}$

$\boxed{\frac{-10+11i}{-13}}$

6.) $(2x^2-4x+7)-4(5-6x^2+3)-(11x^2-3)$

$2x^2-4x+7-20+24x^2-12-11x^2+3$

$\boxed{15x^2-4x-22}$

7.) $(2x-5y)^2$

$(2x-5y)(2x-5y)$

$4x^2-10xy-10xy+25y^2$

$\boxed{4x^2-20xy+25y^2}$

8.) $(5a^3+7)-4(2-6a^2+3a)-4(2a^3-8)$

$5A^3+7-8+24A^2-12A-8A^3+32$

$\boxed{-3A^3+24A^2-12A+31}$

9.) $(3w-2)(2w^2-7w-2)$

$6w^3-21w^2-6w$

$-4w^2+14w+4$

$\boxed{6w^3-25w^2+8w+4}$

10.) Evaluate.

If $f(t)=4t^2-2t+7$; find $f(t+5)$

$f(t+5)=4(t+5)^2-2(t+5)+7$

$=4(t+5)(t+5)-2(t+5)+7$

$=4(t^2+10t+25)-2(t+5)+7$

$=4t^2+40t+100-2t-10+7$

$\boxed{f(t+5)=4t^2+38t+97}$

11.) Evaluate.

If $f(y)=-2y^2-2y$; find $f(y-3)$

$f(y-3)=-2(y-3)^2-2(y-3)$

$=-2(y-3)(y-3)-2(y-3)$

$=-2(y^2-6y+9)-2(y-3)$

$=-2y^2+12y-18-2y+6$

$\boxed{f(y-3)=-2y^2+10y-12}$

Factor Completely.

12.) $2n^4 - 26n^2 + 72$

$2(n^4 - 13n^2 + 36)$

$2(n^2 - 9)(n^2 - 4)$

$\boxed{2(n+3)(n-3)(n+2)(n-2)}$

13.) $27m^3 - 8n^3$

3M 3M 3M · 2N 2N 2N

$\boxed{(3M - 2N)(9M^2 + 6MN + 4N^2)}$

S O AP

14.) $16x^4 - 81$

$(4x^2+9)(4x^2-9)$

$\boxed{(4x^2+9)(2x+3)(2x-3)}$

15.) $6m^3 - 33m^2 + 24m$

$\boxed{3M(2M^2 - 11M + 8)}$

Solve each of the following using the method of your choice.

16.) $-3(2x+4)^2 = 36$

$(2x+4)^2 = -12$

$2x + 4 = \pm\sqrt{-12}$

$2x = -4 \pm 2i\sqrt{3}$

$x = \dfrac{-4 \pm 2i\sqrt{3}}{2}$

$\boxed{x = -2 \pm i\sqrt{3}}$

* $\sqrt{-12}$

$\sqrt{-1}\sqrt{4}\sqrt{3}$

$i \cdot 2 \cdot \sqrt{3}$

$2i\sqrt{3}$

17.) $4x^2 - 5x + 3 = 6x - 4$

28; -7, -4; -11

$4x^2 - 11x + 7 = 0$

$(4x^2 - 7x)(-4x + 7) = 0$

$x(4x-7) - 1(4x-7) = 0$

$(4x-7)(x-1) = 0$

$4x - 7 = 0$ $\quad x - 1 = 0$

$\boxed{x = 7/4}$ $\quad \boxed{x = 1}$

18.) $(2x^4 - 3x^3) + (16x - 24) = 0$

$x^3(2x-3) + 8(2x-3) = 0$

$(2x-3)(x^3+8) = 0$

$(2x-3)(x+2)(x^2-2x+4) = 0$

$2x - 3 = 0$ $\quad \boxed{x = 3/2}$

$x + 2 = 0$ $\quad \boxed{x = -2}$

$x^2 - 2x + 4 = 0$

$x^2 - 2x + \underline{1} = -4 + \underline{1}$

$(x-1)^2 = -3$

$x - 1 = \pm\sqrt{-3}$

$\boxed{x = 1 \pm i\sqrt{3}}$

19.) $c^5 - 125c^2 = 0$

$c^2(c^3 - 125) = 0$

$c^2(c-5)(c^2+5c+25) = 0$

$c^2 = 0$ $\quad \boxed{c = \pm 0}$

$c - 5 = 0$ $\quad \boxed{c = 5}$

$c^2 + 5c + 25 = 0$

$c = \dfrac{-5 \pm \sqrt{(5)^2 - 4(1)(25)}}{2(1)}$

$c = \dfrac{-5 \pm \sqrt{-75}}{2}$

$\boxed{c = \dfrac{-5 \pm 5i\sqrt{3}}{2}}$

20.) Find the remaining zeros for the polynomial given $(x-1)$ is a factor of $P(x) = x^4 - 7x^3 + 17x^2 - 17x + 6$.

1	1	−7	17	−17	6
		1	−6	11	−6
	1	−6	11	−6	0

$x^3 - 6x^2 + 11x - 6 = 0$

USED CALCULATOR

2	1	−6	11	−6
		2	−8	6
	1	−4	3	0

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3 \quad x = 1$

Zeros: 1, 1, 2, 3

21.) Find all roots for the following polynomial, $j(a) = 2a^3 - 2a^2 + 6a - 6$

$(2A^3 - 2A^2) + (6A - 6) = 0$

$2A^2(A-1) + 6(A-1) = 0$

$(A-1)(2A^2+6) = 0$

$A - 1 = 0 \qquad 2A^2 + 6 = 0$

$A = 1 \qquad 2A^2 = -6$

$A^2 = -3$

$A = \pm\sqrt{-3}$

$A = \pm i\sqrt{3}$

Roots: $1, i\sqrt{3}, -i\sqrt{3}$

22.) Solve Algebraically and Graphically.

$y = x^2$

$y + x^2 = 8 \longrightarrow y = -x^2 + 8$

$x^2 + x^2 = 8$

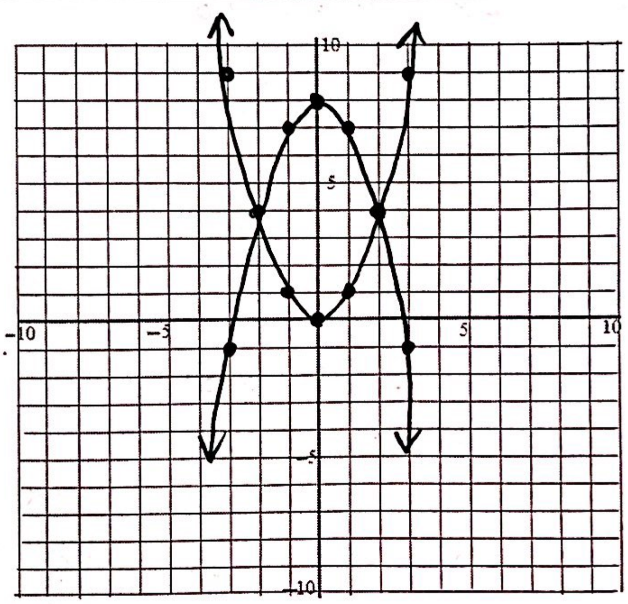
$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

$y = (2)^2 \qquad y = (-2)^2$

$y = 4 \qquad y = 4$



Solutions: (2,4) (−2,4)

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* NEED PLACEHOLDER FOR "$0x^2$"

23.) Find the value of k given that $(x-3)$ is a factor of $p(x) = 2x^3 - kx - 42$.

$$\begin{array}{r|rrrr} 3 & 2 & 0 & -k & -42 \\ & & 6 & 18 & -3k+54 \\ \hline & 2 & 6 & (-k+18) & 0 \end{array}$$

$-3k + 54 - 42 = 0$

$-3k + 12 = 0$

$-3k = -12$

$\boxed{k = 4}$

24.) A grasshopper leaps from a short table (we'll name him J. Cricket) and its path through the air is modeled by the following equations

$h(x) = -x^2 + 12x + 13$ AND $h(x) = -(x-6)^2 + 49$ VERTEX: $(6, 49)$

where height (h) is measured in inches, and horizontal distance traveled (x) is also measured in inches.

a.) How tall is the short table that the grasshopper leaps from?

$h(0) = -(0)^2 + 12(0) + 13$

$= 13$ INCHES

b.) How far has the grasshopper traveled horizontally when it reaches its maximum height?

6 INCHES

c.) What is the maximum height of the grasshopper?

49 INCHES

* YOU CAN USE EITHER EQUATION FOR d, e, f

d.) How high is the grasshopper after it has traveled 3 inches horizontally?

$h(3) = -(3)^2 + 12(3) + 13$

$= -9 + 36 + 13$

$= 40$ IN

$h(3) = -(3-6)^2 + 49$

$= -(-3)^2 + 49$

$= -9 + 49$

$= 40$ IN

e.) How far has the grasshopper traveled horizontally when it is at a height of 33 inches?

$33 = -x^2 + 12x + 13$

$x^2 - 12x + 20 = 0$

$(x-10)(x-2) = 0$

$x = 10 \quad x = 2$

$33 = -(x-6)^2 + 49$

$-16 = -(x-6)^2$

$\sqrt{16} = \sqrt{(x-6)^2}$

$\pm 4 = x - 6$

$x = 6 \pm 4$

$x = 6 + 4 \quad x = 6 - 4$

$x = 10 \quad x = 2$

f.) When the grasshopper lands, how far has it traveled horizontally?

$0 = -x^2 + 12x + 13$

$0 = x^2 - 12x - 13$

$0 = (x-13)(x+1)$

$x - 13 = 0 \quad x + 1 = 0$

$x = 13 \quad x = -1$

$\boxed{13 \text{ IN}}$

$0 = -(x-6)^2 + 49$

$-49 = -(x-6)^2$

$\sqrt{49} = \sqrt{(x-6)^2}$

$\pm 7 = x - 6$

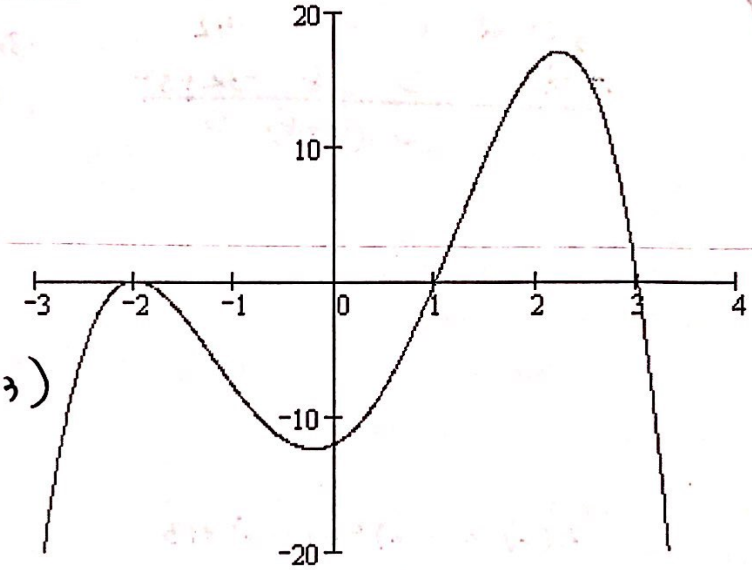
$x = 6 \pm 7$

$x = 6 + 7 \quad x = 6 - 7$

$x = 13 \quad x = -1$

$\boxed{13 \text{ IN}}$

25.) For the given polynomial state the following information.



Degree: Even or Odd

Lead Coefficient: Positive or Negative

Zero(s): $-2, -2, 1, 3$

Factor(s): $(x+2)^2\ (x-1)\ (x-3)$

Equation: $f(x) = -(x+2)^2(x-1)(x-3)$

Rel. Max: $(2.25, 18)\quad(-2, 0)$

Rel. Min: $(-0.25, 12)$

Intervals Increasing: $(-\infty, -2) \cup (-0.25, 2.25)$

Intervals Decreasing: $(-2, -0.25) \cup (2.25, \infty)$

Domain: $(-\infty, \infty)$

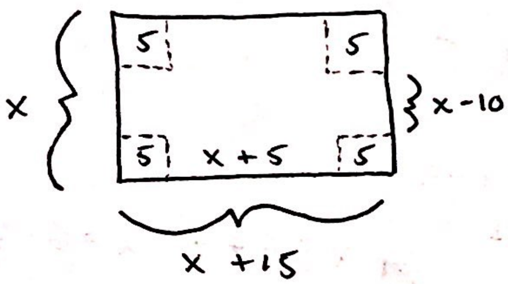
Range: $(-\infty, 18]$

End Behavior:

$as\ x \to \infty, f(x) \to$ $-\infty$

$as\ x \to -\infty, f(x) \to$ $-\infty$

26.) A rectangular piece of cardboard is 15 inches longer than it is wide. If 5-inch squares are cut from each corner, and the remaining piece folded up to form a box, the volume of the box is 1250 cubic inches. Find the dimensions of the piece of cardboard.



$V = l \cdot w \cdot h$

$1250 = (x+5)(x-10)(5)$

$1250 = (x^2 - 10x + 5x - 50)(5)$

$1250 = (x^2 - 5x - 50)(5)$

$1250 = 5x^2 - 25x - 250$

$0 = 5x^2 - 25x - 1500$

$0 = 5(x^2 - 5x - 300)$

$0 = 5(x-20)(x+15)$

$x - 20 = 0 \qquad x + 15 = 0$

$x = 20 \qquad x = -15$

DIMENSIONS

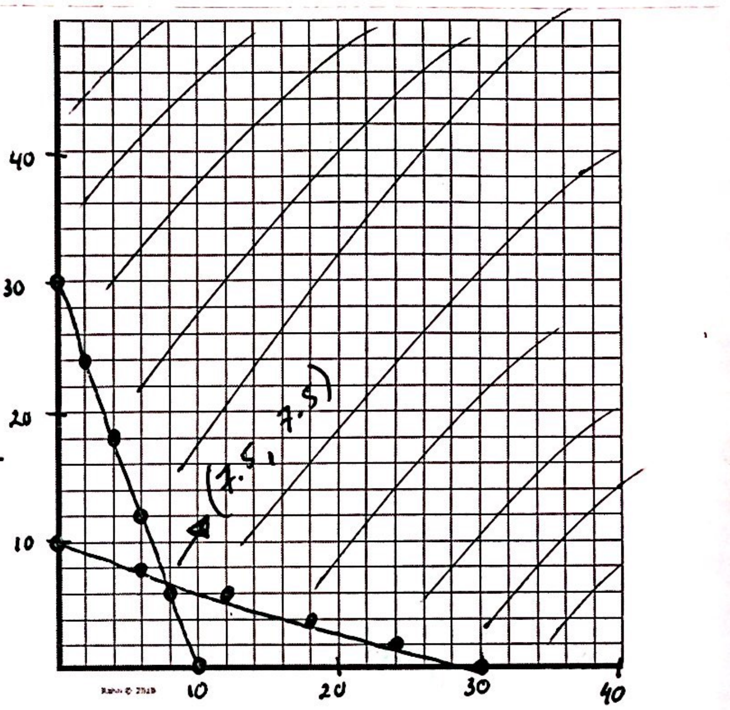
HEIGHT = 5 IN

WIDTH = 10 IN

LENGTH = 25 IN

27.) Shannon's Chocolates produces semisweet chocolate chips and milk chocolate chips at its plants in Wichita, KS and Moore, OK. The Wichita plant produces 3000 pounds of semisweet chips and 2000 pounds of milk chocolate chips each day at a cost of $1000, while the Moore plant produces 1000 pounds of semisweet chips and 6000 pounds of milk chocolate chips each day at a cost of $1500. Shannon has an order from Food Box Supermarkets for at least 30,000 pounds of semisweet chips and 60,000 pounds of milk chocolate chips. How should Shannon schedule its production so that it can fill the order at minimum cost? What is the minimum cost?

let x = number of days of production at Wichita plant.
let y = number of days of production at Moore plant.



Constraints

$x \geq 0$

$y \geq 0$

$3000x + 1000y \geq 30{,}000 \longrightarrow y \geq -3x + 30$

$2000x + 6000y \geq 60{,}000 \longrightarrow y \geq -\tfrac{1}{3}x + 10$

Objective Function

$f(x,y) = 1000x + 1500y$

Vertices

$(0,30) = 45{,}000$

$(7.5, 7.5) = 18{,}750$

$(30,0) = 30{,}000$

Answer

7.5 DAYS OF PRODUCTION AT WICHITA PLANT

7.5 DAYS OF PRODUCTION AT MOORE PLANT

TO MINIMIZE COST AT $18,750

Simplify.

28.) $\sqrt{20x^5y^{11}}$

$\sqrt{4x^4y^{10}} \cdot \sqrt{5xy}$

$\boxed{2x^2y^5\sqrt{5xy}}$

29.) $\left(\frac{36x^4}{5y^6}\right)^{-\frac{3}{2}}$

$\frac{36^{-3/2} \; x^{-12/2}}{5^{-3/2} \; y^{-18/2}}$

$\frac{5^{3/2} \; y^{9}}{36^{3/2} \; x^{6}}$

$\boxed{\frac{5\sqrt{5}y^9}{216x^6}}$

$* \; 36^{3/2}$

$(\sqrt{36})^3$

$(6)^3$

216

$* \; 5^{3/2}$

$\sqrt{5^3}$

$\sqrt{5 \cdot 5 \cdot 5}$

$5\sqrt{5}$

30.) $(-2x^3y^{-4})(5x^2y^3)^2$

$(-2x^3y^{-4})(25x^4y^6)$

$\boxed{-50x^7y^2}$

Solve.

31.) $4 - \sqrt[4]{2x+4} = 2$

$-\sqrt[4]{2x+4} = -2$

$\left[\sqrt[4]{2x+4}\right]^4 = [2]^4$

$2x+4 = 16$

$2x = 12$

$\boxed{x = 6}$

32.) $(3x+4)^{\frac{2}{3}} - 6 = 10$

$\left[(3x+4)^{2/3}\right]^{3/2} = [16]^{3/2}$

$* (\sqrt{16})^3$

$(4)^3$

64

$3x+4 = \pm 64$

$3x = -4 \pm 64$

$x = \dfrac{-4 \pm 64}{3}$

$x = \dfrac{-4+64}{3}$ $\quad$ $x = \dfrac{-4-64}{3}$

$\boxed{x = \dfrac{60}{3}}$ $\quad$ $\boxed{x = \dfrac{-68}{3}}$

Find the inverse function.

33.) $f(x) = \sqrt[3]{2x+5}$

$x = \sqrt[3]{2y+5}$

$x^3 = 2y+5$

$x^3 - 5 = 2y$

$\boxed{\dfrac{x^3-5}{2} = y}$

34.) $g(x) = \sqrt[4]{(x-1)^3}$

$x = \sqrt[4]{(y-1)^3}$

$x^4 = (y-1)^3$

$\sqrt[3]{x^4} = y - 1$

$\boxed{\sqrt[3]{x^4} + 1 = y}$

OR

$x = \sqrt[4]{(y-1)^3}$

$[x]^{4/3} = \left[(y-1)^{3/4}\right]^{4/3}$

$x^{4/3} = y - 1$

$\boxed{x^{4/3} + 1 = y}$

For the following problems, let $f(x) = 3x - 4$ and let $g(x) = x^2 + 6$

35.) Find $f(g(x))$

$= 3(x^2+6)-4$

$= 3x^2+18-4$

$\boxed{= 3x^2+14}$

36.) Find $g(f(x))$

$= (3x-4)^2+6$

$= 9x^2-24x+16+6$

$\boxed{= 9x^2-24x+22}$

37.) $f(g(2))$

$g(2) = (2)^2+6$

$= 4+6$

$= 10$

$f(10) = 3(10)-4$

$= 30-4$

$\boxed{= 26}$

38.) Identify the translations on the following functions.

a. $f(x) = 3\sqrt{x+2} - 5$

VERTICAL STRETCH OF 3; HORIZONTAL SHIFT LEFT 2 UNITS; VERTICAL SHIFT DOWN 5

b. $g(x) = (x-4)^2 - 10$

HORIZONTAL SHIFT OF 4 UNITS RIGHT; VERTICAL SHIFT OF 10 UNITS DOWN