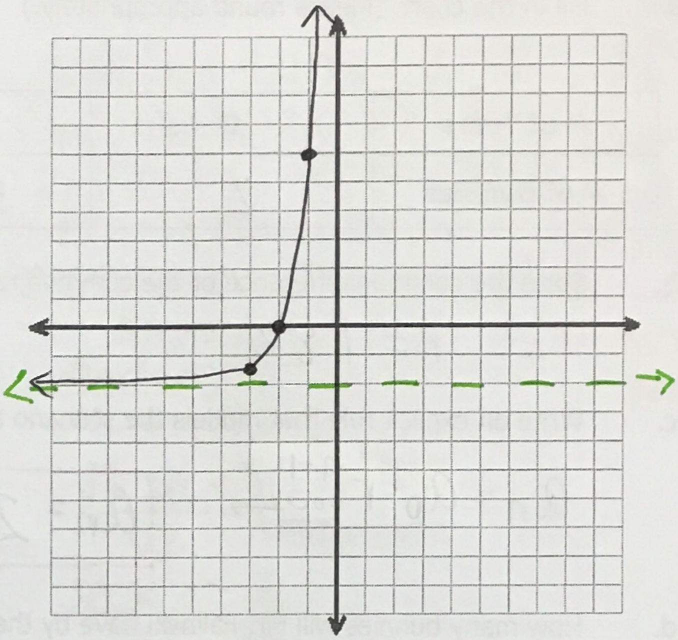
Algebra 2
Semester 2
Finals Review #1

Name: Key

Date: ______________ Per: ______

Graph the exponential function and find all the information listed below.

1.) $f(x) = \frac{1}{2}(4)^{x+3} - 2$



Parent Function: $f(x) = \frac{1}{2}(4)^x$

Growth / Decay: growth

Asymptote: $y = -2$

Domain: $(-\infty, \infty)$

Range: $(-2, \infty)$

End Behavior:

As $x \to -\infty, f(x) \to$ -2

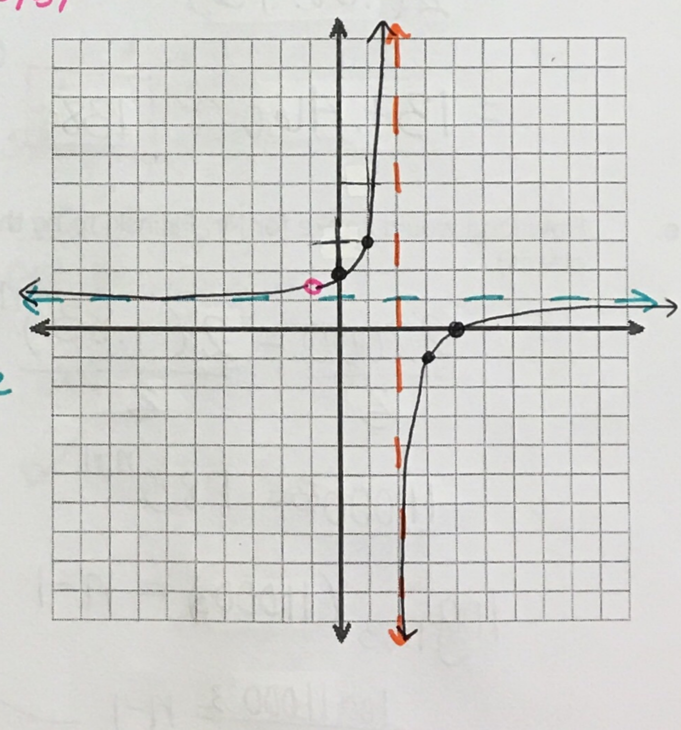
As $x \to \infty, f(x) \to$ ∞

Describe the transformation(s), in the order they are made:

* left 3 units
* down 2 units

Hole @ $x=-1$
$(-1, 5/3)$

2.) $f(x) = \frac{x^2-3x-4}{x^2-x-2} \to \frac{(x-4)\cancel{(x+1)}}{(x-2)\cancel{(x+1)}}$



y-intercept: $(0, 2)$

Hole(s): $(-1, 5/3)$

Zero(s): $x = 4$

Vertical Asymptote(s): $x = 2$ *denominator

Horizontal Asymptote(s): $y = 1$ *same degree top to bottom

Domain: $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

Range: $(-\infty, 1) \cup (1, \infty)$

End Behavior:

as $x \to \infty, f(x) \to$ 2

as $x \to -\infty, f(x) \to$ 2

3.) Mr. Falinski has decided to purchase two pet bunnies for each of his two children. His two daughters each receive a boy bunny and a girl bunny. Not knowing the severe risk of owning multiple bunnies, let alone keeping them in the same cage, Mr. Falinski is now the proud owner of a rapidly increasing population of bunnies. Mr. Falinski was able to calculate (go figure) that the bunnies were growing by a rate of 83% every year.

a. Fill in the chart. (Please round appropriately.) ↳ $r = 1.83$

# of Years	0	1	2	3
# of Bunnies	2	3.66 ↳ 3	6.70 ↳ 6	12.26 ↳ 12

b. State the common difference or the common ratio. (Include the appropriate symbol.)

$r = 1.83$

c. Write an explicit rule that models the scenario above. *Geometric

$a_n = a_1(1.83)^{n-1} \rightarrow \boxed{a_n = 3(1.83)^{n-1}}$

d. How many bunnies will Mr. Falinski have by the 8th year?

$a_8 = 3(1.83)^{8-1}$

$= 3(1.83)^7$

$= 3(68.73)$

$= 206.20 \approx \boxed{206}$

Bunnies: 206

e. How long would it take for Mr. Falinski to be the care taker of 22,000 bunnies? (round to three decimal places)

$\frac{22000}{3} = \frac{3(1.83)^{n-1}}{3}$

$7333.33 = 1.83^{n-1}$

$\log_{1.83}(7333.33) = n-1$

$\frac{\log 7333.33}{\log 1.83} = n-1$

$14.73 = n-1$

$15.73 = n$

$\boxed{n = 16 \text{ years}}$

Time: 16 years

ve for all values of x over the interval of $[0, 2\pi)$. * Don't stress about #4-7

S^+	A^+
T^+	C^+

Answers in radians

4.) $3\cos^2 x - 5\cos x + 2 = 0$

$3x^2 - 5x + 2 = 0$ (6; −3, −2; −5)

$(3x^2 - 3x)(-2x + 2) = 0$

$3x(x-1) - 2(x-1) = 0$

$(3x-2)(x-1) = 0$

$3\cdot\cos x - 2 = 0$ / $+2 \quad +2$

$\frac{3\cdot\cos x}{3} = \frac{2}{3} \rightarrow \cos x = 2/3$ ← Next year

$\cos x - 1 = 0$ / $+1 \quad +1$

$\cos x = 1$

$\cos x = 1$ at $\boxed{0 \,\&\, 2\pi}$

5.) $4\cos^2 x - 4\sin x = 1$

$4(1 - \sin^2 x) - 4\sin x = 1$

$4 - 4\sin^2 x - 4\sin x - 1 = 0$

$4\sin^2 x + 4\sin x - 3 = 0$ (−12; 6, −2; 4)

$(4x^2 + 6x)(-2x - 3) = 0$

$2x(2x+3) - 1(2x+3) = 0$

$(2x-1)(2x+3) = 0$

$2\sin x - 1 = 0 \qquad 2\sin x + 3 = 0$

$\sin x = 1/2 \qquad \sin x = -3/2$ ← Next year

$\sin x = \frac{1}{2}$ at $\boxed{\frac{\pi}{6} \,\&\, \frac{5\pi}{6}}$

6.) $4\sin^2 x - 2 = 0$

$+2 \; +2$

$\frac{4\sin^2 x}{4} = \frac{2}{4}$

$\sqrt{\sin^2 x} = \sqrt{\frac{1}{2}}$

$\sin x = \frac{\pm 1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\sin x = \pm\frac{\sqrt{2}}{2}$ at $\boxed{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \,\&\, \frac{7\pi}{4}}$

7.) $\sqrt{3}\sec^2 x = 2\sec x$

$\frac{\sqrt{3}\sec^2 x}{\sec x} = \frac{2\sec x}{\sec x}$

$\frac{\sqrt{3}\sec x}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\sec x = \frac{2}{\sqrt{3}}$

$\downarrow$

$\cos x = \frac{\sqrt{3}}{2}$ at $\boxed{\frac{\pi}{6} \,\&\, \frac{11\pi}{6}}$

Prove each trigonometric identity.

8.) $\frac{1+\cot^2\theta}{\sec^2\theta} = \cot^2\theta$

$\frac{\csc^2\theta}{\sec^2\theta}$

$\frac{\frac{1}{\sin^2\theta}}{\frac{1}{\cos^2\theta}}$

$\frac{1}{\sin^2\theta} \cdot \frac{\cos^2\theta}{1}$

$\frac{\cos^2\theta}{\sin^2\theta}$

✓ $\cot^2\theta =$

9.) $\csc^2\theta \cdot \tan^2\theta - 1 = \tan^2\theta$

$\frac{1}{\cancel{\sin^2\theta}} \cdot \frac{\cancel{\sin^2\theta}}{\cos^2\theta} - 1$

$\frac{1}{\cos^2\theta} - 1$

$\sec^2\theta - 1$

✓ $\tan^2\theta =$

Use the properties of Logarithms to expand the following problem.

10.) $\log_3 3x^2\sqrt{5}$

$\log_3 3 + \log_3 x^2 + \log_3 \sqrt{5}$

$\boxed{\log_3 3 + 2\log_3 x + \frac{1}{2}\log_3 5}$

11.) $\log \frac{2x^2y}{5z^3}$

$\log(2x^2y) - \log(5z^3)$

$[\log 2 + \log x^2 + \log y] - [\log 5 + \log z^3]$

$\boxed{[\log 2 + 2\log x + \log y] - [\log 5 + 3\log z]}$

Use the properties of Logarithms to condense the following problem.

12.) $3\log_2 x + 2\log_2 5 - \log_2 20 - 4\log_2 y$

$\log_2 x^3 + \log_2 5^2 - \log_2 20 - \log_2 y^4$

$\boxed{\log_2 \frac{25x^3}{20y^4}}$

13.) $\frac{1}{3}[\log_4(x-2) - 3\log_4 2]$

$\frac{1}{3}[\log_4(x-2) - \log_4 2^3]$

$\frac{1}{3}[\log_4 \frac{(x-2)}{8})$

$\boxed{\log_4 \sqrt[3]{\frac{(x-2)}{8}}}$

Solve each equation. If necessary, round answers to 4 decimal places.

14.) $\cancel{\log_3}(5m+1) = \cancel{\log_3}(m^2-49)$

$5m+1 = m^2-49$
$-5m \ -1 \qquad -5m \ -1$

$m^2 - 5m - 50 = 0$ -50; -10, 5; -5

$(m-10)(m+5) = 0$

$\boxed{m=10}$ $\cancel{m=-5}$ extraneous

15.) $\left(\frac{1}{4}\right)^{x-1} = 32^{x+3}$

$(2^{\boxed{-2}})^{\boxed{x-1}} \boxed{=} (2^{\boxed{5}})^{\boxed{x+3}}$

$-2(x-1) = 5(x+3)$

$-2x + 2 = 5x + 15$
$+2x \ -15 \quad +2x \ -15$

$\frac{-13}{7} = \frac{7x}{7}$

$\boxed{x = -\frac{13}{7}}$

16.) $3^{2x^2} \cdot 3^{5x} = 27$

$3^{\boxed{2x^2+5x}} \boxed{=} 3^{\boxed{3}}$

$2x^2 + 5x = 3$
$-3 \quad -3$

$2x^2 + 5x - 3 = 0$ -6; -1, 6; 5

$(2x^2 - x)(+6x - 3) = 0$

$x(2x-1) + 3(2x-1) = 0$

$(2x-1)(x+3) = 0$

$\boxed{x = \frac{1}{2} \text{ or } x = -3}$

17.) $2 + 7\log_{11}(3m-8) = 16$
$-2 \qquad\qquad -2$

$\frac{7 \cdot \log_{11}(3m-8)}{7} = \frac{14}{7}$

$\log_{11}(3m-8) = 2$

$11^2 = 3m-8$

$121 = 3m - 8$
$+8 \qquad +8$

$\frac{129}{3} = \frac{3m}{3}$

$\boxed{m = 43}$

.) $\log_2(2-2x)+\log_2(1-x)=5$

$\log_2((2-2x)(1-x))=5$

$2^5 = 2-2x-2x+2x^2$

$32 = 2-4x+2x^2$

$2x^2-4x-30=0$

$2(x^2-2x-15)=0$

$2(x-5)(x+3)=0$

$\cancel{x=5}$ extraneous

$\boxed{x=-3}$

19.) $4e^{2x-1}+5=21$

$-5 \quad -5$

$\frac{4\cdot e^{2x-1}}{4} = \frac{16}{4}$

$e^{2x-1}=4$

$\ln 4 = 2x-1$

$1.39 = 2x-1$

$\frac{2.39}{2} = \frac{2x}{2} \rightarrow \boxed{x=1.19}$

20.) $\log(3x^2+2x-4)=0$

$10^0 = 3x^2+2x-4$

$1 = 3x^2+2x-4$

$0 = 3x^2+2x-5$

-15; -3, 5; 2

$(3x^2-3x)(+5x-5)$

$3x(x-1)+5(x-1)=0$

$(x-1)(3x+5)=0$

$\boxed{x=1 \text{ or } x=-5/3}$

21.) $-4+7\ln(2n-1)=31$

$+4 \quad +4$

$\frac{7\cdot\ln(2n-1)}{7} = \frac{35}{7}$

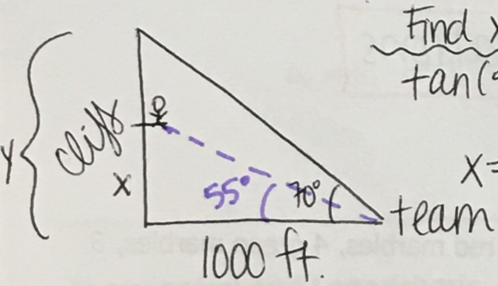
$\ln(2n-1)=5$

$e^5 = 2n-1$

$148.41 = 2n-1$

$\frac{149.41}{2} = \frac{2n}{2} \longrightarrow \boxed{n=74.71}$

22.) A rescue team 1000 ft. away from the base of a vertical cliff measures the angle of elevation to the top of the cliff to be 70°. A climber is stranded on a ledge. The angle of elevation from the rescue team to the ledge is 55°. How far is the stranded climber from the top of the cliff?



Find x

$\tan(55) = \frac{x}{1000}$

$x = 1428.15$

Find y

$\tan(70) = \frac{y}{1000}$

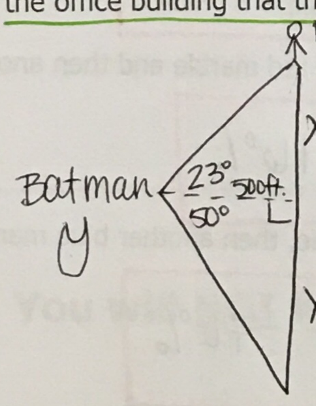
$y = 2747.48$

$2747.48 - 1428.15$

$= 1319.33$

$\boxed{1319.33 \text{ ft away}}$

23.) Batman is on the second floor of an apartment building. He spots the Riddler 500 ft. away on top of an office building. Batman takes out his handy dandy "Bat-culator" and measure the angle of elevation for the top of the office building is 23° and the angle of depression for the base of the building is 50°. How tall is the office building that the Riddler is standing on?



Find x

$\tan(23°) = \frac{x}{500}$

$x = 212.24$

Find y

$\tan(50°) = \frac{y}{500}$

$y = 595.88$

$212.24 + 595.88$

$= 808.12$

$\boxed{808.12 \text{ ft. tall}}$

24.) For a population that grows at a constant rate of $r\%$ per year, the formula $P(t) = p_0(1+r)^t$ models the population t years after an initial population of p_0 is counted. The population of the city of Stapletown was 455,000 in 2008. Assume the population grows at a constant rate of 2% per year. What would the population of Stapletown be in 2014? → 2014 − 2008 = 6

$P(t) = 455{,}000(1+.02)^t$

$P(6) = 455{,}000(1.02)^6$

$455{,}000(1.13) = 512{,}403.9 = \boxed{512{,}403 \text{ people}}$

25.) If \$1000 is invested at 5% annual interest compounded continuously. How long will it take for the amount to triple?

↳ $A = P \cdot e^{rt}$

$A = 1000 \cdot e^{.05t}$

$\frac{3000}{1000} = \frac{1000 \cdot e^{.05t}}{1000}$

$3 = e^{.05t}$

$\ln 3 = .05t$

$1.10 = .05t$

$t = 21.97 \rightarrow \boxed{22 \text{ years}}$

26.) Radio station call letters consist of four letters, the first of which much be a **K** or a **W**. If the letters can be repeated, how many such radio stations are possible?

$\underline{2} \cdot \underline{2} \cdot \underline{26} \cdot \underline{26} = \boxed{2704 \text{ radio stations}}$

27.) Maddie was so excited to find a random bag on the floor that contained 8 red marbles, 4 green marbles, 3 blue marbles, and 5 yellow marbles. → Total 20 marbles

a.) What is the probability that Maddie reaches in the bag and selects a green marble and then a yellow marble if she doesn't replace the first marble?

$\frac{4}{20} \cdot \frac{5}{19} = \frac{20}{380} = \boxed{\frac{1}{19} \text{ or } 5.3\%}$

b.) What is the probability that Maddie reaches in the bag and selects a red marble and then another red marble if she replaces the first marble in the bag?

$\frac{8}{20} \cdot \frac{8}{20} = \frac{64}{400} = \boxed{\frac{4}{25} \text{ or } 16\%}$

c.) What is the probability that Maddie reaches in a selects a blue marble, then another blue marble, and then a red marble if she doesn't replace the marbles?

$\frac{3}{20} \cdot \frac{2}{19} \cdot \frac{8}{18} = \frac{48}{6840} = \boxed{\frac{2}{285} \text{ or } .70\%}$