Algebra 2
Chapter 7 Review

$\log_{name}(name)^x =$ Key

Date: _____ **Period:** _____

Rewrite each equation in exponential form.

1.) $\log_9 27 = \frac{3}{2}$

$9^{3/2} = 27$

2.) $\log_8 4 = \frac{2}{3}$

$8^{2/3} = 4$

3.) $\log_{\frac{1}{4}} 64 = -3$

$\left(\frac{1}{4}\right)^{-3} = 64$

Rewrite each equation in logarithmic form.

4.) $4 = \left(\frac{1}{2}\right)^{-2}$

$\log_{\frac{1}{2}} 4 = -2$

5.) $10^{-2} = 0.01$

$\log 0.01 = -2$

6.) $\frac{1}{32} = 2^{-5}$

$\log_2 \frac{1}{32} = -5$

Expand each logarithm.

7.) $\ln \frac{a^2 b^3}{c^4}$

$\ln a^{(2)} + \ln b^{(3)} - \ln c^{(4)}$

$2\ln a + 3\ln b - 4\ln c$

8.) $\log_8 8a^5\sqrt{3}$

$\log_8 8 + \log_8 a^{(5)} + \log_8 \sqrt{3}$

$1 + 5\log_8 a + \frac{1}{2}\log_8 3$

9.) $\log\left(\frac{2\sqrt{x}}{5}\right)^3$

$3[\log 2 + \log x^{1/2} - \log 5]$

$3[\log 2 + \frac{1}{2}\log x - \log 5]$

Condense each logarithm.

10.) $(x)\log_4 m + (\frac{1}{4})\log_4 n - \log_4 p$

$\log_4 m^x + \log_4 \sqrt[4]{n} - \log_4 p$

$\log_4 \frac{m^x \sqrt[4]{n}}{p}$

11.) $\frac{1}{2}(\ln 4 + \ln x) - (3)\ln y$

$(\frac{1}{2})(\ln 4x) - \ln y^3$

$\ln \frac{\sqrt{4x}}{y^3}$

12.) $(3)\log_4 4 - (2)\log_4 5 - (5)\log_4 x$

$\log_4 4^3 - \log_4 5^2 - \log_4 x^5$

$\log_4 \frac{4^3}{5^2 x^5}$ OR

$\log_4 \frac{64}{25x^5}$

13.) $\ln(2x+5) + \ln(4x-3)$

$\ln[(2x+5)(4x-3)]$

$(2x+5)(4x-3)$

$8x^2 - 6x + 20x - 15$

$8x^2 + 14x - 15$

OR

$\ln(8x^2 + 14x - 15)$

Solve.

14.) $49^{2x-3} = 343^{2x-6}$

$(7^2)^{2x-3} = (7^3)^{2x-6}$

$4x-6 = 6x-18$

$-4x \quad -4x$

$-6 = 2x-18$

$+18 \quad +18$

$\frac{2x}{2} = \frac{12}{2}$ $\boxed{x=6}$

15.) $2^x + 13 = 35$

$-13 \quad -13$

$2^x = 22$

$\log_2 22 = x$

$\frac{\log 22}{\log 2} = x \rightarrow \boxed{x = 4.459}$

16.) $\frac{7e^{2x}}{7} = \frac{17.5}{7}$

$e^{2x} = 2.5$

$\ln 2.5 = 2x$

$\frac{.9162}{2} = \frac{2x}{2}$

$\boxed{x = .458}$

17.) $36^{x-3} = 216^{6-2x}$

$(6^2)^{x-3} = (6^3)^{6-2x}$

$2x-6 = 18-6x$

$+6x \quad +6x$

$8x-6 = 18$

$\frac{8x}{8} = \frac{24}{8}$ $\boxed{x=3}$

18.) $\log_2(x) + \log_2(2x+3) = \log_2 9$

$\log_2(x(2x+3)) = \log_2 9$

$2x^2+3x = 9$

$2x^2+3x-9 = 0$

-18, -3, 6, 3

$(2x^2-3x)(+6x-9) = 0$

$x(2x-3)+3(2x-3) = 0$

$(2x-3)(x+3) = 0$

$\boxed{x=3/2}$ $x=-3$ extraneous

19.) $\log_6 3x = 2$

$6^2 = 3x$

$\frac{36}{3} = \frac{3x}{3}$

$\boxed{x=12}$

20.) $\log(5x) + \log(x-3) = 2$

$\log(5x(x-3)) = 2$

$10^2 = 5x^2-15x$

$5x^2-15x-100 = 0$

$5(x^2-3x-20) = 0$

$a=1 \quad b=-3 \quad c=-20$

$x = \frac{3 \pm \sqrt{(-3)^2-4(1)(-20)}}{2(1)}$

$\boxed{x=6.217}$

$x=-3.217$ extraneous

21.) $\ln(x+1) - \ln(x-2) = \ln x$

$\ln \frac{(x+1)}{(x-2)} = \ln x$

$\frac{x+1}{x-2} = \frac{x}{1}$

$x(x-2) = x+1$

$x^2-2x = x+1$

$x^2-3x-1 = 0$

$a=1, \quad b=-3, \quad c=-1$

$x = \frac{3 \pm \sqrt{(-3)^2-4(1)(-1)}}{2(1)}$

$\boxed{x=3.303}$

$x=-.303$ extraneous

22.) In 1990, the fan population of Chicago Bulls was 18,233. As Michael Jordan lead the Bulls to three straight NBA Championship Titles over the next three years, the fan population grew 23.4% each of those years. What was the fan population in 1994? → 1994 − 1990 = 4 years

$A = 18233(1+.234)^t$

$18233(1.234)^4 = 42278.42 \rightarrow$ **42278 people**

23.) On the day you were born, a deposit of \$50,000 is made in a trust fund that pays 8.75% interest, compounded continuously. $A = 50{,}000 \cdot e^{.0875t}$

a.) Find the balance on your 35th birthday.

$A = 50{,}000 \cdot e^{.0875 \cdot 35}$

$A = 50{,}000 \cdot e^{3.0625} \rightarrow$ **\$1,069,047.14**

b.) How long would you have to wait for the balance in the trust fund to double?

$\frac{100{,}000}{50{,}000} = \frac{50{,}000 \cdot e^{.0875t}}{50{,}000}$

$2 = e^{.0875t}$

$\ln 2 = .0875t$

$\frac{0.6931}{.0875} = \frac{.0875t}{.0875} \rightarrow$ **t = 7.92 years OR 8 years**

24.) The antler spread a (in inches) and shoulder height h (in inches) of an adult male American Elk are related by the model $h = 116\log(a + 40) - 176$. Approximate the shoulder height of a male American Elk with an antler spread of 55 inches.

$h = 116 \cdot \log(55+40) - 176$

h = 53.42 inches

25.) The speed of the wind S (in meters per second) near the edge of a tornado and the radius r (in meters) of the tornado at ground level are related by the equation $S = 65 + 93\log r$. On March 18, 1925, a large tornado struck portions of Missouri, Illinois, and Indiana with a wind speed at the edge of about 283 meters per second. Approximate the radius of this tornado at ground level.

$283 = 65 + 93\log r$

$-65 \quad -65$

$\frac{218}{93} = \frac{93\log r}{93}$

$2.344 = \log r$

$10^{2.344} = r$

r = 220.84 meters

26.) After t years, the value V of a car that originally cost \$14000 is given by $V = 14{,}000(0.75)^t$.

a.) Find the value of the car 2 years after it was purchased.

$V = 14000(0.75)^2\rightarrow$ **\$7875.00**

b.) Approximate how long it would take for the value of the car to be \$6500.

$6500 = 14{,}000(0.75)^t \rightarrow 0.75^t = 0.46 \rightarrow \log_{0.75} 0.46 = t$

t = 2.7 years

27.) You have discovered another new radioactive element, and you have decided to name it after your favorite class!!! Algebrite!!! After careful study, you have determined that the amount of Algebrite remaining from an original sample of 45 grams is governed by the formula:

$$A = 45e^{-0.0142t}$$

A) How long would it take for the element Algebrite to decay to 14 grams?

$\frac{14}{45} = \frac{45e^{-0.0142t}}{45}$

$.311 = e^{-0.0142t}$

$\ln .311 = -0.0142t$

$\frac{-1.168}{-0.0142} = \frac{-0.0142t}{-0.0142}$

$t = 82.226$

t = 82 years

28.) Carbon-14 is commonly used in dating objects because it has a large half-life. A half-life is the amount of time it takes for half (aka 50%) of an element to decay. Suppose you have a sample of 100 grams and the equation representing the decay is given by

$$A = 100e^{-0.003t}$$

A) What is the half-life (in years) of Carbon-14? (How long for half a sample to decay?)

$\frac{50}{100} = \frac{100 \cdot e^{-.003t}}{100}$

$0.50 = e^{-.003t}$

$\ln .5 = -.003t$

$\frac{-.6931}{-.003} = \frac{-.003t}{-.003}$

$t = 231.05$

231 years

29.) You deposited some money in an account that pays 2.25% interest, compounded continuously. How long will it take your money to double? *try with any amount.

$A = \_\_ \cdot e^{.0225t}$

$\frac{200}{100} = \frac{100 \cdot e^{.0225t}}{100}$

$2 = e^{.0225t}$

$\ln 2 = .0225t$

$\frac{.6931}{.0225} = \frac{.0225t}{.0225}$

$t = 30.81$

About 31 years

30.) How much money would have after 9 years, if you deposited $12,000 into an account that has an interest of 4.25% and is compounded annually? $A = 12000(1+r)^t$

$A = 12000(1+.0425)^9$

A = $17452.83

31.) In 2 years, you are planning to purchase the new TI-84 Super-Duper Plus 3D Calculator with all the top notch functions that costs $225. How much should you invest into an account yielding 6.5% interest, compounded continuously.

$225 = P \cdot e^{.065 \cdot 2}$

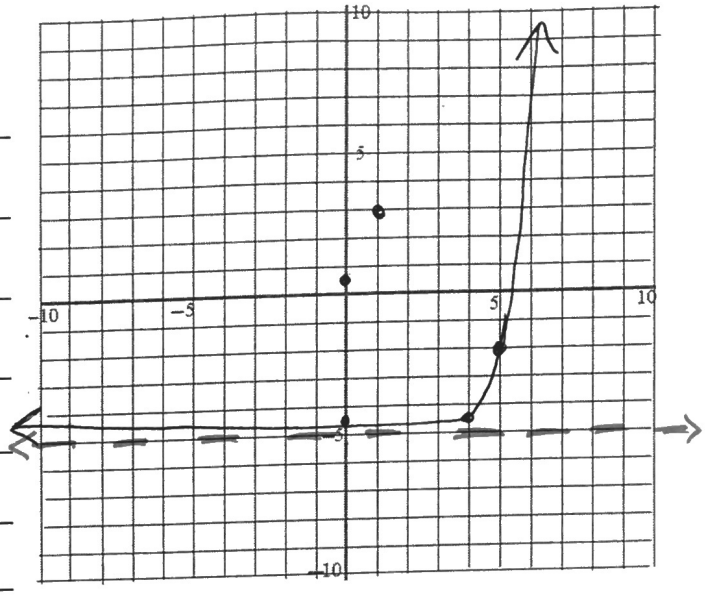
$225 = P \cdot e^{.13}$

$\frac{225}{1.1388} = \frac{P \cdot 1.1388}{1.1388}$

$P = 197.57 \rightarrow$ **$198**

Graph. Identify all the important.

32.) $y = \left(\frac{1}{2}\right)(6)^{x-4} - 5$



Parent Function: $y = \frac{1}{2}(6)^x$

Growth/Decay: growth

Asymptote: $y = -5$

Transformations: Right 4
down 5

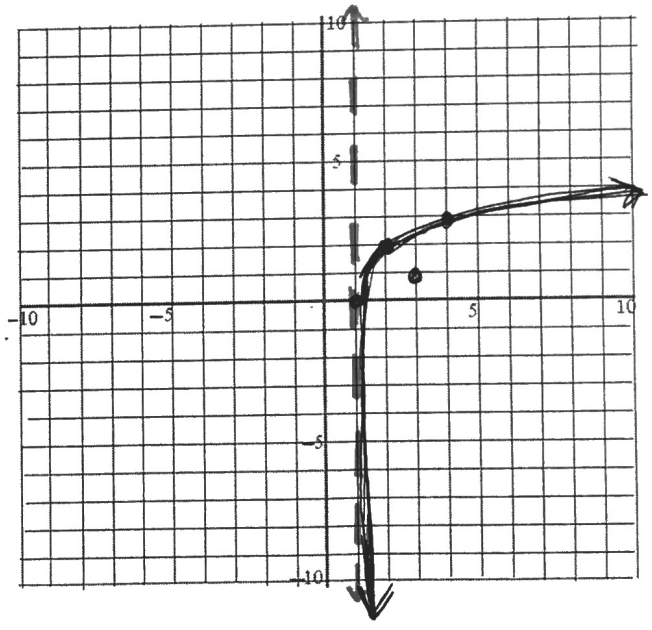
Domain: $(-\infty, \infty)$

Range: $(-5, \infty)$

End Behavior: $as\ x \to \infty, f(x) \to \infty$

$as\ x \to -\infty, f(x) \to -5$

33.) $y = \log_3(x-1) + 2$



Parent Function: $y = \log_3(x)$

Asymptote: $x = 1$

Transformations: Right 1
up 2

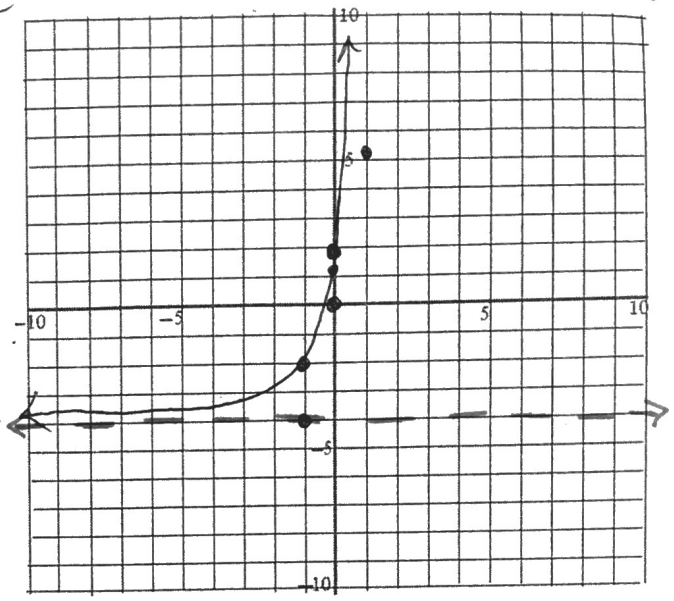
Domain: $(1, \infty)$

Range: $(-\infty, \infty)$

End Behavior: $as\ x \to \infty, f(x) \to \infty$

$as\ x \to -\infty, f(x) \to$ DNE

34.) $y = 2e^{x+1} - 4$ parent function: $y = 2 \cdot e^x$



Growth or Decay: growth

Transformations: left 1

down 4

Domain: $(-\infty, \infty)$

Range: $(-4, \infty)$

End Behavior: as $x \to \infty, f(x) \to \infty$

as $x \to -\infty, f(x) \to -4$

Find the inverse for each of the given functions.

33.) $y = (3)\left(\frac{2}{3}\right)^{x+4} - 3$

$x = 3 \cdot \frac{2}{3}^{y+4} - 3$

$+3 \qquad +3$

$\frac{x+3}{3} = \frac{3 \cdot \frac{2}{3}^{y+4}}{3}$

$\frac{x+3}{3} = \frac{2}{3}^{y+4}$

$\log_{\frac{2}{3}} \frac{x+3}{3} = y + 4$

$-4 \qquad -4$

$y = \log_{\frac{2}{3}} \frac{x+3}{3} - 4$

34.) $y = e^{2x+1} - 3$

$x = e^{2y+1} - 3$

$+3 \qquad +3$

$x + 3 = e^{2y+1}$

$\ln(x+3) = 2y + 1$

$-1 \qquad -1$

$\frac{\ln(x+3) - 1}{2} = \frac{2y}{2}$

$y = \frac{\ln(x+3) - 1}{2}$