

Properties of Logarithms – Expanding & Condensing
Honors Algebra 2

N A M E: _____ KEY
Date: _____ Period: _____

Expand.

1.) $\log_4 5x^3y$

$$\log_4 5 + \log_4 x^3 + \log_4 y$$

$$\log_4 5 + 3\log_4 x + \log_4 y$$

2.) $\ln \frac{\sqrt{3x-5}}{7}$

$$\ln \sqrt{3x-5} - \ln 7$$

$$\ln (3x-5)^{1/2} - \ln 7$$

$$\frac{1}{2} \ln (3x-5) - \ln 7$$

3.) $\ln \frac{x^2}{y^2 z^3}$

$$\ln x^2 - \ln y^2 z^3$$

$$\ln x^2 - (\ln y^2 + \ln z^3)$$

$$2 \ln x - (2 \ln y + 3 \ln z)$$

or $2 \ln x - 2 \ln y - 3 \ln z$

4.) $\ln \sqrt{\frac{x^2}{y^3}}$

$$\ln \left(\frac{x^2}{y^3} \right)^{1/2}$$

$$\frac{1}{2} \left[\ln \left(\frac{x^2}{y^3} \right) \right]$$

$$\frac{1}{2} [\ln x^2 - \ln y^3]$$

5.) $\log \frac{y^4 \sqrt{x}}{z^4}$

$$\log y^4 \sqrt{x} - \log z^4$$

$$\log y^4 + \log x^{1/2} - \log z^4$$

$$4 \log y + \frac{1}{2} \log x - 4 \log z$$

6.) $\log_2 \frac{xy^5}{z^2}$

$$\log_2 xy^5 - \log_2 z^2$$

$$\log_2 x + \log_2 y^5 - \log_2 z^2$$

$$\log_2 x + 5 \log_2 y - 2 \log_2 z$$

7.) $\log_3 \left(\frac{3x^2}{yz} \right)$

$$\log_3 3x^2 - \log_3 yz$$

$$\log_3 3 + \log_3 x^2 - (\log_3 y + \log_3 z)$$

$$1 + 2 \log_3 x - (\log_3 y + \log_3 z)$$

$$\text{Condensed: } x^2 - \log_3 y - \log_3 z$$

8.) $\ln \left(\frac{\sqrt[3]{xy}}{t^{4/3}} \right)$

$$\ln (xy)^{1/3} - \ln t^{4/3}$$

$$\frac{1}{3} [\ln xy] - \frac{4}{3} \ln t$$

$$\frac{1}{3} [\ln x + \ln y] - \frac{4}{3} \ln t$$

$$\frac{1}{3} \ln x + \frac{1}{3} \ln y - \frac{4}{3} \ln t$$

9.) $\log \frac{(x+1)^2(x^2+1)}{90xy}$

$$\log (x+1)^2 (x^2+1) - \log 90xy$$

$$\log (x+1)^2 + \log (x^2+1) - (\log 90 + \log x + \log y)$$

$$2 \log (x+1) + \log (x^2+1) - (\log 90 + \log x + \log y)$$

or

$$2 \log (x+1) + \log (x^2+1) - \log 90 + \log x - \log y$$

10.) $\frac{1}{2} [3 \ln x - \ln(x+1) - \ln(x-1)]$

$$\frac{1}{2} [\ln x^3 - \ln(x+1) - \ln(x-1)]$$

$$\frac{1}{2} \left[\ln \frac{x^3}{(x+1)(x-1)} \right]$$

$$\ln \sqrt{\frac{x^3}{(x+1)(x-1)}}$$

11.) $3 \log_4 x + 2 \log_4 3 + \frac{1}{2} \log_4 y$

$$\log_4 x^3 + \log_4 3^2 + \log_4 y^{1/2}$$

$$\log_4 9x^3 \sqrt{y}$$

12.) $2 \log 9 + \log(3x-5)$

$$\log 9^2 + \log(3x-5)$$

$$\log 81(3x-5)$$

13.) $\frac{1}{3} [2 \ln(x+3) + \ln x - \ln(x^2-1)]$

$$\frac{1}{3} [\ln(x+3)^2 + \ln x - \ln(x^2-1)]$$

$$\frac{1}{3} \left[\ln \frac{x(x+3)^2}{(x^2-1)} \right]$$

$$\ln \left(\frac{x(x+3)^2}{(x^2-1)} \right)^{1/3} \text{ or } \ln \sqrt[3]{\frac{x(x+3)^2}{(x^2-1)}}$$

14.) $2 \ln 8 - 5 \ln 2$

$$\ln 8^2 - \ln 2^5$$

$$\ln \frac{8^2}{2^5}$$

$$\ln \frac{64}{32}$$

$$\ln 2$$

16.) $\frac{1}{2} \log x - \log y + \log z - \frac{1}{3} \log w$

$$\log x^{1/2} - \log y + \log z - \log w^{1/3}$$

$$\log \frac{\sqrt{x} \cdot z}{y^{3/\sqrt{w}}}$$

15.) $\log(2x - 3) + \log(3x - 4)$

$$\log (2x-3)(3x-4)$$

or

$$\log 6x^2 - 17x + 12$$

17.) $\ln e - 2(\ln x^2 + \ln y)$

$$\ln e - 2(\ln x^2 + y)$$

$$\ln e - \ln (x^2 y)^2$$

$$1 - \ln (x^2 y)^2$$

Retro Question:

13.) $y = 3\left(\frac{1}{2}\right)^{x-4} - 3$

Parent Function: $y = \left(\frac{1}{2}\right)^x$

Growth/Decay: DECAY

Domain: $(-\infty, \infty)$

Range: $(-3, \infty)$

Asymptote: $y = -3$

End Behavior:

AS $x \rightarrow -\infty$, $f(x) \rightarrow \infty$

AS $x \rightarrow \infty$, $f(x) \rightarrow -3$

