Verifying Trig Identities

Pythagorean IDs

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

1.) Work with one side at a time (generally the more complicated side)
2.) You may multiply one side by a special form of 1, such as sin(x)/sin(x)
3.) ***Don't*** add/subtract/divide/multiply **both** sides by anything
4.) Look for opportunities to factor or combine fractions
5.) Look for chances to use identities
6.) Change everything to sine and cosine
7.) TRY SOMETHING!

1.) $\tan^3 x = \tan x \sec^2 x - \tan x$

- $= \tan x(\sec^2 x - 1)$
- ★ $= \tan x(\tan^2 x)$
- $\tan^3 x = \tan^3 x$

★ since $\tan^2 x + 1 = \sec^2 x$
$\tan^2 x = \sec^2 x - 1$

2.) $\dfrac{\tan^2\theta}{1+\sec\theta} = \dfrac{1-\cos\theta}{\cos\theta}$

- $\dfrac{(\sec^2\theta - 1)}{1+\sec\theta} =$
- $\dfrac{(\sec\theta+1)(\sec\theta-1)}{(1+\sec\theta)} =$
- $\sec\theta - 1 =$
- $\dfrac{1}{\cos\theta} - 1 =$
- $\dfrac{1}{\cos\theta} - \dfrac{\cos\theta}{\cos\theta} =$
- $\dfrac{1-\cos\theta}{\cos\theta} = \dfrac{1-\cos\theta}{\cos\theta}$

3.) $\dfrac{\sec^2 x - 1}{\sec^2 x} = \sin^2 x$

- $\dfrac{\sec^2 x}{\sec^2 x} - \dfrac{1}{\sec^2 x} =$
- $1 - \cos^2 x =$
- ★ $\sin^2 x = \sin^2 x$

★ since $\sin^2 x + \cos^2 x = 1$
$\sin^2 x = 1 - \cos^2 x$

4.) $(\sec^2\alpha - 1)(\sin^2\alpha - 1) = -\sin^2\alpha$

- $(\tan^2\alpha)(-\cos^2\alpha) =$
- $\left(\dfrac{\sin^2\alpha}{\cancel{\cos^2\alpha}}\right)\left(\dfrac{-\cancel{\cos^2\alpha}}{1}\right) =$
- $-\sin^2\alpha = -\sin^2\alpha$

5.) $\csc x - \sin x = \cos x \cot x$

- $\dfrac{1}{\sin x} - \dfrac{\sin x}{1} =$
- $\dfrac{1}{\sin x} - \dfrac{\sin^2 x}{\sin x} =$
- $\dfrac{1-\sin^2 x}{\sin x} =$
- $\dfrac{\cos^2 x}{\sin x} =$
- $\cos x \cdot \dfrac{\cos x}{\sin x} =$
- $\cos x \cot x =$

6.) $\csc A + \cot A = \dfrac{\sin A}{1-\cos A}$

- $\dfrac{1}{\sin A} + \dfrac{\cos A}{\sin A} =$
- $\dfrac{1+\cos A}{\sin A} =$
- $\dfrac{(1-\cos A)(1+\cos A)}{(1-\cos A)\sin A} =$
- $\dfrac{1-\cos^2 A}{\sin A(1-\cos A)} =$
- $\dfrac{\sin^2 A}{\sin A(1-\cos A)} =$
- $\dfrac{\sin A}{1-\cos A} = \dfrac{\sin A}{1-\cos A}$

7.) $\dfrac{1}{1-\cos\beta} + \dfrac{1}{1+\cos\beta} = 2\csc^2\beta$

- $\dfrac{(1+\cos\beta)+(1-\cos\beta)}{(1-\cos\beta)(1+\cos\beta)} =$
- $\dfrac{2}{1-\cos^2\beta} =$
- $\dfrac{2}{\sin^2\beta} =$
- $2\csc^2\beta = 2\csc^2\beta$

8.) $\dfrac{\cot^2\theta}{1+\csc\theta} = \dfrac{1-\sin\theta}{\sin\theta}$

- $\dfrac{\csc^2\theta - 1}{1+\csc\theta} =$
- $\dfrac{(\csc\theta+1)(\csc\theta-1)}{(1+\csc\theta)} =$
- $\csc\theta - 1 =$
- $\dfrac{1}{\sin\theta} - 1 =$
- $\dfrac{1}{\sin\theta} - \dfrac{\sin\theta}{\sin\theta} =$
- $\dfrac{1-\sin\theta}{\sin\theta} = \dfrac{1-\sin\theta}{\sin\theta}$

9.) $\sec^2 x + \tan^2 x \sec^2 x = \sec^4 x$

- $\sec^2 x\,(1+\tan^2 x) =$
- $\sec^2 x\,(\sec^2 x) =$
- $\sec^4 x \quad = \sec^4 x$

10.) $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$

- $(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) =$
- $(1)\ (\cos^2 x - \sin^2 x) =$
- $\cos^2 x - \sin^2 x =$
- $(1-\sin^2 x) - \sin^2 x =$
- $1 - 2\sin^2 x = 1 - 2\sin^2 x$

11.) $\frac{1}{\sin(x)\cos(x)} - \frac{\cos(x)}{\sin(x)} = \tan(x)$

- $\frac{1}{\sin x \cos x} - \frac{\cos^2 x}{\sin x \cos x} =$
- $\frac{1-\cos^2 x}{\sin x \cos x} =$
- $\frac{\sin^2 x}{\sin x \cos x} =$
- $\frac{\sin x}{\cos x} =$
- $\tan x = \tan x$

12.) $\frac{\sin(x)}{\csc(x)} + \frac{\cos(x)}{\sec(x)} = 1$

- $\sin x \cdot \frac{1}{\csc x} + \cos x \cdot \frac{1}{\sec x} =$
- $\sin x \sin x + \cos x \cos x =$
- $\sin^2 x + \cos^2 x =$
- $1 = 1$

13.) $\frac{1}{1+\cos(x)} = csc^2(x) - \csc(x)\cot(x)$

- $= \frac{1}{\sin^2 x} - \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}$
- $= \frac{1}{\sin^2 x} - \frac{\cos x}{\sin^2 x}$
- $= \frac{1-\cos x}{\sin^2 x}$
- $= \frac{1-\cos x}{1-\cos^2 x}$
- $= \frac{1-\cos x}{(1+\cos x)(1-\cos x)}$
- $\frac{1}{1+\cos x} = \frac{1}{1+\cos x}$

14.) $\sin^3 x \cos^2 x = \sin^3 x - \sin^5 x$

- $= \sin^3 x\,(1-\sin^2 x)$
- $= \sin^3 x\,(\cos^2 x)$
- $\sin^3 x \cos^2 x = \sin^3 x \cos^2 x$

15.) $\sec^2 x + \csc^2 x = \sec^2 x \csc^2 x$

- $\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} =$
- $\frac{\sin^2}{\cos^2 x \sin^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x} =$
- $\frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} =$
- $\frac{1}{\cos^2 x \sin^2 x} =$
- $\frac{1}{\cos^2 x} \cdot \frac{1}{\sin^2 x} =$
- $\sec^2 x \csc^2 x = \sec^2 x \csc^2 x$