

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

1.)  $x^3 - x^2 - 5x - 3; x + 1$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -5 & -3 \\ & & -1 & 2 & 3 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

$x^2 - 2x - 3$

$(x-3)(x+1)$

2.)  $6x^3 - 25x^2 + 2x + 8; x - 4$

$$\begin{array}{r|rrrr} 4 & 6 & -25 & 2 & 8 \\ & \downarrow & 24 & -4 & -8 \\ \hline & 6 & -1 & -2 & 0 \end{array}$$

$6x^2 - x - 2$

$(3x-2)(2x+1)$

3.)  $16x^5 - 32x^4 - 81x + 162; x - 2$

$$\begin{array}{r|rrrrrr} 2 & 16 & -32 & 0 & 0 & -81 & 162 \\ & \downarrow & 32 & 0 & 0 & 0 & -162 \\ \hline & 16 & 0 & 0 & 0 & -81 & 0 \end{array}$$

$16x^4 - 81$

$(4x^2 - 9)(4x^2 + 9)$

$(2x-3)(2x+3)(4x^2+9)$

4.)  $x^4 + 2x^3 - 8x - 16; x + 2$

$$\begin{array}{r|rrrrr} -2 & 1 & 2 & 0 & -8 & -16 \\ & \downarrow & -2 & 0 & 0 & 16 \\ \hline & 1 & 0 & 0 & -8 & 0 \end{array}$$

$x^3 - 8$

$(x-2)(x^2+2x+4)$

Find the value of k (algebraically) so that each remainder is 3.

5.)  $(x^2 + kx - 17) \div (x - 2)$

$$\begin{array}{r|rrr} 2 & 1 & k & -17 \\ & \downarrow & 2 & 2(k+2) \\ \hline & 1 & k+2 & 3 \end{array}$$

$2(k+2) - 17 = 3$

$2(k+2) = 20$

$k+2 = 10$

$k = 8$

6.)  $(x^2 + 5x + 7) \div (x + k)$

$$\begin{array}{r|rrr} -k & 1 & 5 & 7 \\ & \downarrow & -k & k^2 - 5k \\ \hline & 1 & 5-k & 3 \end{array}$$

$k^2 - 5k + 7 = 3$

$k^2 - 5k + 4 = 0$

$(k-4)(k-1) = 0$

$k = 4$   $k = 1$

Given a function and one of its zeros, find all of the remaining zeros of the function.

7.)  $f(x) = x^3 - 4x^2 + 6x - 4$ ; zero: 2

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 6 & -4 \\ & \downarrow & 2 & -4 & 4 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$$x^2 - 2x + 2 = 0$$

$$x = \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2}$$

$$x = \frac{2 \pm \sqrt{4 - 8}}{2}$$

$$x = \frac{2 \pm \sqrt{-4}}{2}$$

$$x = \frac{2 \pm \sqrt{-4}}{2}$$

$$x = \frac{2 \pm 2i}{2}$$

$$x = 1 \pm i$$

8.)  $f(x) = x^3 - 13x^2 + 55x - 91$ ; zero: 7

$$\begin{array}{r|rrrr} 7 & 1 & -13 & 55 & -91 \\ & \downarrow & 7 & -42 & 91 \\ \hline & 1 & -6 & 13 & 0 \end{array}$$

$$x^2 - 6x + 13 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 4(1)(13)}}{2}$$

$$x = \frac{6 \pm \sqrt{-16}}{2}$$

$$x = \frac{6 \pm 4i}{2}$$

$$x = 3 \pm 2i$$

State the number of possible zeros. Then find all of the zeros of each function.

9.)  $f(x) = x^3 - 3x^2 - 10x + 24$

# of Zeros: 3

$$\begin{array}{r|rrrr} -3 & 1 & -3 & -10 & 24 \\ & \downarrow & -3 & +18 & -24 \\ \hline & 1 & -6 & 8 & 0 \end{array}$$

$$x^2 - 6x + 8 = 0$$

$$(x - 4)(x - 2) = 0$$

$$x = 4 \quad x = 2$$

Zeros:  $-3, 2, 4$

10.)  $f(x) = x^4 - 6x^2 + 12x - 8$

# of Zeros: 4

Zeros:  $1, 24, -3, 24$

11.)  $f(x) = x^3 - 7x^2 + 37x - 75$

# of Zeros: 3

$$\begin{array}{r|rrrr} 3 & 1 & -7 & 37 & -75 \\ & \downarrow & 3 & -12 & 75 \\ \hline & 1 & -4 & 25 & 0 \end{array}$$

$$x^2 - 4x + 25 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4(1)(25)}}{2}$$

$$x = \frac{4 \pm \sqrt{16 - 100}}{2}$$

$$x = \frac{4 \pm \sqrt{-84}}{2}$$

$$x = \frac{4 \pm 2i\sqrt{21}}{2}$$

$$x = 2 \pm i\sqrt{21}$$

$$x = \frac{4 \pm \sqrt{-84}}{2}$$

$$x = \frac{4 \pm \sqrt{-84}}{2}$$

Zeros:  $3, 2 \pm i\sqrt{21}$

12.)  $f(x) = 2x^5 - x^4 - 2x + 1$

# of Zeros: 5

$$\begin{array}{r|rrrrr} -1 & 2 & -1 & 0 & 0 & -2 & 1 \\ & \downarrow & -2 & 3 & -3 & +3 & -1 \\ \hline & 2 & -3 & 3 & -3 & +1 & 0 \end{array}$$

$$2x^4 - 3x^3 + 3x^2 - 3x + 1 = 0$$

$$\begin{array}{r|rrrrr} 1 & 2 & -3 & 3 & -3 & 1 \\ & \downarrow & 2 & -1 & 2 & -1 \\ \hline & 2 & -1 & 2 & -1 & 0 \end{array}$$

$$2x^3 - x^2 + 2x - 1 = 0$$

$$(2x^3 - x^2) + (2x - 1) = 0$$

$$x^2(2x - 1) + 1(2x - 1)$$

$$(2x - 1)(x^2 + 1) = 0$$

$$2x - 1 = 0$$

$$x = 1/2$$

$$x^2 + 1 = 0$$

$$x^2 = -1$$

$$x = \pm i$$

Zeros:  $-1, 1, 1/2, \pm i$