Honors Algebra 2
The Polynomial Review

Name: KEY
Date: ______ Period: ____

1.) What is the remainder when $p(x) = x^6 - 2x^3 + x - 1$ is divided by $(x + 1)$?

$$\begin{array}{r|rrrrrrr} -1 & 1 & 0 & 0 & -2 & 0 & 1 & -1 \\ & \downarrow & -1 & 1 & -1 & 3 & -3 & 2 \\ \hline & 1 & -1 & 1 & -3 & 3 & -2 & 1 \end{array}$$

a.) -3
b.) -1
$\boxed{\text{c.) } 1}$
d.) 3

2.) If $p(x) = x^3 - 2x^2 + 9x - 2$, which of the following statement(s) is/are true?

i. $x - 3$ is a factor of $p(x)$
ii. $x = 3$ is a root of $p(x)$
iii. $p(3) = 34$
iv. $p(-3) = 34$

$$\begin{array}{r|rrrr} 3 & 1 & -2 & 9 & -2 \\ & \downarrow & 3 & 3 & 36 \\ \hline & 1 & 1 & 12 & 34 \end{array}$$

a.) i only
$\boxed{\text{b.) iii only}}$
c.) i and ii only
d.) i and iii only
e.) i and iv only

3.) How many real roots must the following equation have?

$$x^4(x^2 - 4) + 9(x^2 - 4) = 0$$

- $(x^2-4)(x^4+9) = 0$

2 REAL 4 IMAGINARY

a.) 1
$\boxed{\text{b.) } 2}$
c.) 4
d.) none

4.) Determine the quotient when $x^3 - 2x^2 - 9$ is divided by $(x - 3)$?

$$\begin{array}{r|rrrr} 3 & 1 & -2 & 0 & -9 \\ & \downarrow & 3 & 3 & 9 \\ \hline & 1 & 1 & 3 & 0 \end{array}$$

$x^2 + x + 3$

a.) $x^2 + 5x + 15$
b.) $x^2 + x - 6$
c.) $x^2 - 5x + 6$
$\boxed{\text{d.) } x^2 + x + 3}$

5.) What are the zeros of the polynomial function $f(x) = 2x^3 - 8x^2 + 6x$?

$\boxed{\text{a.) } x = 0, 1, 3}$
b.) $x = 1, 2, 3$
c.) $x = 0, -1, -3$
d.) $x = 0, 1, -4$

- $0 = 2x(x^2 - 4x + 3)$
- $0 = 2x(x-3)(x-1)$
- $2x = 0 \quad x - 3 = 0 \quad x - 1 = 0$

$x = 0 \quad x = 3 \quad x = 1$

6.) If $(x+2)$ is a root of $3x^3 + kx^2 - 31x - 54$. What is the value of k?

$$\begin{array}{r|rrrr} -2 & 3 & k & -31 & -54 \\ & \downarrow & -6 & -2k+12 & 4k+38 \\ \hline & 3 & (k-6) & -2k-19 & 0 \end{array}$$

$4k - 16 = 0$

$4k = 16$

$k = 4$

a.) -8
b.) -4
c.) -51
$\boxed{\text{d.)} \quad 4}$

7.) Find the remainder when $f(x) = x^6 + 5x^5 - x^3 + x - 6$ is divided by $(x+1)$.

$$\begin{array}{r|rrrrrrr} -1 & 1 & 5 & 0 & -1 & 0 & 1 & -6 \\ & \downarrow & -1 & -4 & 4 & -3 & 3 & -4 \\ \hline & 1 & 4 & -4 & 3 & -3 & 4 & -16 \end{array}$$

a.) 0
$\boxed{\text{b.)} \quad -10}$
c.) -1
d.) -12

8.) Given that a function $f(x)$ has a zero at $x = 3$ with multiplicity 2, then we know that...

a.) the graph of $f(x)$ crosses the y – axis at 3.
b.) $as\ x \to \infty, f(x) \to \infty$
c.) the graph of $f(x)$ crosses the x – axis at 3.
$\boxed{\text{d.)}}$ the graph of $f(x)$ touches but does not cross the x – axis at 3.

9.) The polynomials $p(x) = x^4 + 5x^3 - 2x^2 - 24x$ has a zero at $x = 2$. Factor p completely.

$(x-2)$

$$\begin{array}{r|rrrr} 2 & 1 & 5 & -2 & -24 \\ & \downarrow & 2 & 14 & 24 \\ \hline & 1 & 7 & 12 & 0 \end{array}$$

$x^3 + 7x^2 + 12x$

$x(x^2 + 7x + 12)$

$\to x(x+4)(x+3)$

a.) $p(x) = x(x+2)(x+3)(x+4)$
b.) $p(x) = (x-2)(x-3)(x-4)$
c.) $p(x) = x(x+2)(x-3)(x-4)$
$\boxed{\text{d.)} \quad p(x) = x(x-2)(x+3)(x+4)}$

10.) For the given polynomials function, $f(x) = -5x^2(x-8)(x+2)^3$, find the zeros of the function and state the multiplicity of each.

$x = 0 \quad x = 8 \quad x = -2$

a.) $-2, multiplicity\ 1;\ \ 2, multiplicity\ 1;\ \ 8, multiplicity\ 1$
b.) $-2, multiplicity\ 3;\ \ 0, multiplicity\ 2;\ \ 8, multiplicity\ 1;\ \ 2, multiplicity\ 1$
c.) $-2, multiplicity\ 1;\ \ 0, multiplicity\ 2;\ \ 8, multiplicity\ 1$
$\boxed{\text{d.)}}$ $-2, multiplicity\ 3;\ \ 0, multiplicity\ 2;\ \ 8, multiplicity\ 1$

11.) For the given polynomials function, $f(x) = x^3 + 6x^2 - x - 6$, find the zeros of the function and state the multiplicity of each.

$0 = x^2(x+6) - 1(x+6)$

$0 = (x+6)(x^2-1)$

$0 = (x+6)(x+1)(x-1)$

$x = -6 \quad x = -1 \quad x = 1$

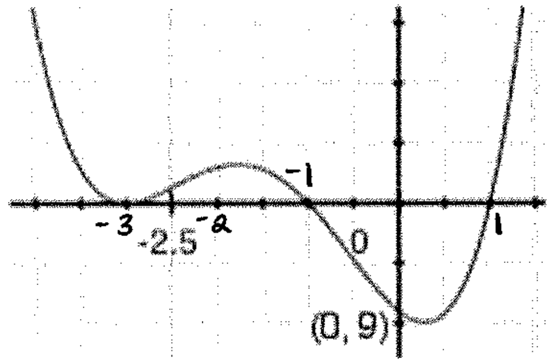
a.) $-1, multiplicity\ 1;\ \ 1, multiplicity\ 1;\ \ 6, multiplicity\ 1$
b.) $-6, multiplicity\ 2;\ \ 1, multiplicity\ 1$
$\boxed{\text{c.)}}$ $-6, multiplicity\ 1;\ \ -1, multiplicity\ 1;\ \ 1, multiplicity\ 1$
d.) $-6, multiplicity\ 3;\ \ -1, multiplicity\ 1;\ \ 1, multiplicity\ 1$

12.) The equation that would best model the following graph is:

$x = -3$ (e)
$x = -1$
$x = 1$

$\boxed{\text{a.)} \quad f(x) = (x+3)^2(x^2-1)}$
b.) $f(x) = (x+3)(x-1)(x+1)$
c.) $f(x) = x^4 + 6x^3 + 9x^2 - 6x - 9$
d.) $f(x) = -(x+3)^2(x^2-1)$

- $(x+3)^2(x+1)(x-1)$
- $(x+3)^2(x^2-1)$



13.) The function $f(x)$ has a zero of 2 with a multiplicity 3. We know...

a.) Since 3 is an odd number, the graph touches but does not cross the x – axis.
$\boxed{\text{b.)}}$ Since 3 is an odd number, the graph crosses the x – axis.
c.) Since 2 is an even number, the graph touches but does not cross the x – axis.
d.) Since 2 is an even number, the graph crosses the x – axis.

14.) The function $f(x)$ has a zero of 3 with a multiplicity 2. We know...

a.) Since the zero is 3, the graph crosses the y – axis at 3.
b.) Since the zeros is 3, the graph goes up to the right.
c.) Since the multiplicity is 2, the graph crosses the x – axis.
$\boxed{\text{d.)}}$ Since the multiplicity is 2, the graph touches but does not cross the x – axis.

15.) Using a graphing calculator, find the relative minimum, relative maximum, and zeros of $f(x) = 3x^3 + 15x^2 - 12x - 60$. If necessary, round to the nearest hundredth.

a.) $\textit{relative minimum}: (-62.24, 0.36), \textit{relative maximum}: (37.79, -3.69), \textit{zeros}: \; x = 5, -2, 2$
$\boxed{\text{b.)}}$ $\textit{relative minimum}: (0.36, -62.24), \textit{relative maximum}: (-3.70, 37.79), \textit{zeros}: \; x = -5, -2, 2$
c.) $\textit{relative minimum}: (0.36, -62.24), \textit{relative maximum}: (-3.70, 37.79), \textit{zeros}: \; x = 5, -2$
d.) $\textit{relative minimum}: (-62.24, 0.36), \textit{relative maximum}: (37.79, -3.69), \textit{zeros}: \; x = -5, -2$

16.) Using the polynomial, $f(x) = -2x^3 + 4x - 8$, explain how the degree and leading coefficient will affect the end behavior.

$\boxed{\text{a.)}}$ Because the degree is odd, the ends will point in opposite direction, and because the leading coefficient is negative the graph will point down on the right.

b.) Because the degree is odd, the ends will point in opposite direction, and because the leading coefficient is negative the graph will point up on the right.

c.) Because the degree is odd, the ends will point in the same direction, and because the leading coefficient is negative the graph will point down on the right.

d.) Because the degree is odd, the ends will point in the same direction, and because the leading coefficient is negative the graph will point up on the right.

17.) **Divide**

$$\begin{array}{r} x^2 + 6x - 12 \\ x^2+4 \overline{\smash{)}\, x^4+6x^3-8x^2+5x-8} \\ -(x^4+0x^3+4x^2) \\ \hline 6x^3-12x^2+5x \\ -(6x^3+0x^2+24x) \\ \hline -12x^2-19x-8 \\ -(-12x^2+0x-48) \\ \hline -19x+40 \end{array}$$

$$\boxed{x^2+6x-12+\frac{-19x+40}{x^2+4}}$$

18.) **Find ALL zeros of the given function.**

a.) $f(x) = x^3 + 4x^2 + 14x + 20$

$-\quad +\quad -\quad +$

Total # of Zeros: 3

Number of Positive Real Zeros	Number of Negative Real Zeros	Number of Imaginary Zeros
0	3	0
0	1	2

$$\begin{array}{r|rrrr} -2 & 1 & 4 & 14 & 20 \\ & \downarrow & -2 & -4 & -20 \\ \hline & 1 & 2 & 10 & 0 \end{array}$$

$\left(\frac{2}{2}\right)^2$

$(1)^2$

1

- $x^2+2x+10=0$
- $x^2+2x+\underline{1}=-10+\underline{1}$
- $x^2+2x+1=-9$
- $(x+1)^2=-9$
- $x+1=\pm 3i$
- $x=-1\pm 3i$

Zeros: $-2,\ -1\pm 3i$

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b.) $f(x) = x^4 - 6x^3 + 25x^2 - 96x + 144$

$+\quad +\quad +\quad +\quad +$

Total # of Zeros: 4

Number of Positive Real Zeros	Number of Negative Real Zeros	Number of Imaginary Zeros
4	0	0
2	0	2
0	0	4

$$\begin{array}{r|rrrrr} 3 & 1 & -6 & 25 & -96 & 144 \\ & \downarrow & 3 & -9 & 48 & -144 \\ \hline & 1 & -3 & 16 & -48 & 0 \end{array}$$

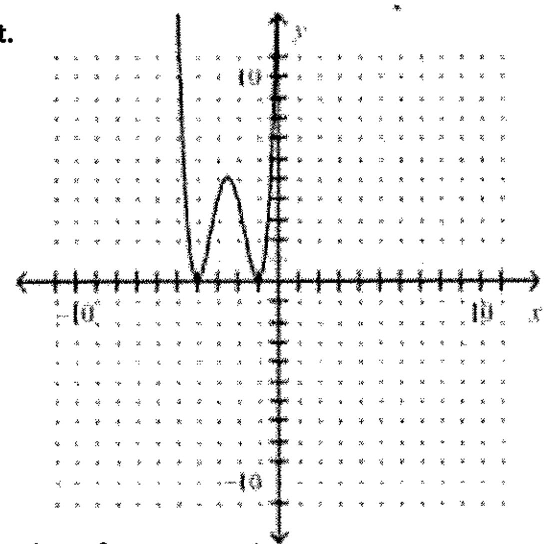
- $x^3-3x^2+16x-48=0$
- $x^2(x-3)+16(x-3)=0$
- $(x-3)(x^2+16)=0$
- $x-3=0 \qquad x^2+16=0$

$x=3 \qquad x^2=-16$

$x=\pm 4i$

Zeros: $3,\ 3,\ 4i,\ -4i$

19.) A complete graph of a polynomial function g is shown at the right.



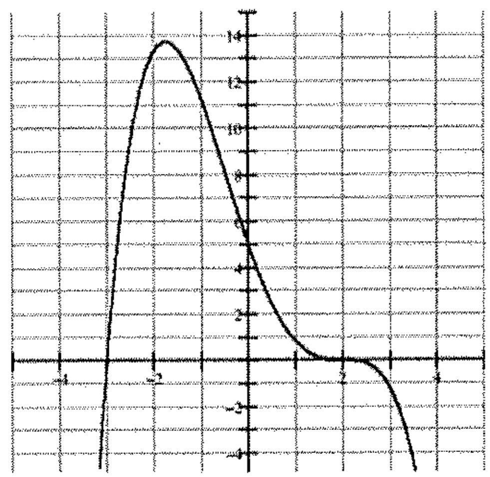
a.) Is the degree of $g(x)$ even or odd? EVEN.
Explain: END BEHAVIOR FOR BOTH IS GOING TOWARD $+\infty$, BOTH GOING IN SAME DIRECTION

b.) Is the leading coefficient of $g(x)$ positive or negative? POSITIVE.
Explain: END BEHAVIOR FOR BOTH IS GOING TOWARD $+\infty$

c.) What do the real zeros of $g(x)$ appear to be? -1, -4 (REPEATED)

d.) What is the smallest possible degree of $g(x)$? 4. Explain: GRAPH BOUNCES AT -1 AND -4 INDICATING THERE IS AN EVEN NUMBER OF ZEROS SO THE SMALLEST POSSIBLE DEGREE WOULD BE 4 ASSUMING WE HAVE $(x+1)^2(x+4)^2$ AS FACTORS.

20.) Write the polynomial function of lowest degree in factored form for the following graph.



ZEROS: -3, 2

FACTORS: $(x+3)(x-2)$

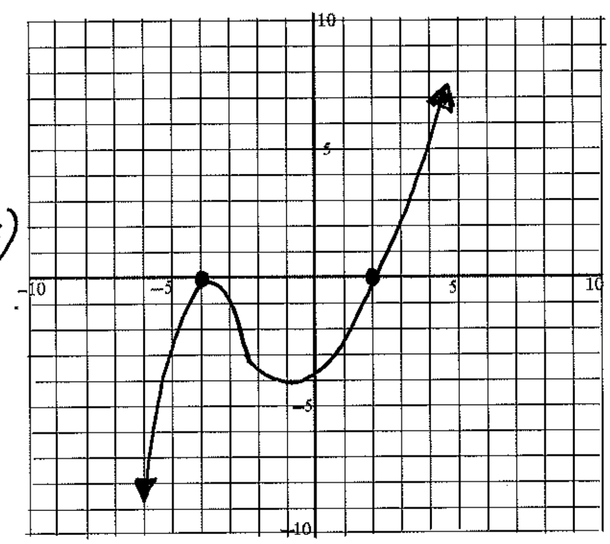
$f(x) = (x+3)(x-2)$

21.) Using what you know about zeros, multiplicity, and end behavior draw a sketch of the graph of the following function:

$f(x) = 3(x-2)^3(x+4)^2$

ZEROS: 2 (MULTIPLICITY OF 3 - PASS)

-4 (MULTIPLICITY OF 2 - BOUNCE)



DEGREE: 5 (ODD)

L.C.: POSITIVE

END BEHAVIOR:

22.) Find all roots for $f(x) = x^3 + x^2 - 4x + 6$ given $(x+3)$ is a factor of the polynomial.

$$\begin{array}{r|rrrr} -3 & 1 & 1 & -4 & 6 \\ & \downarrow & -3 & 6 & -6 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$\left(\frac{-2}{2}\right)^2$ $x^2 - 2x + 2 = 0$

$(-1)^2$ $x^2 - 2x + \underline{1} = -2 + \underline{1}$

1 $(x-1)^2 = -1$

$x - 1 = \pm i$

$x = 1 \pm i$

ROOTS : $-3, 1+i, 1-i$

23.) Sketch $f(x) = x^3 + 6x^2 - x - 6$.

$x^2(x+6) - 1(x+6) = 0$

$(x+6)(x^2-1) = 0$

$(x+6)(x-1)(x+1) = 0$

$x = -6 \quad x = 1 \quad x = -1$

Degree: 3 (ODD)

Lead Coefficient: 1 (POSITIVE)

End Behavior: *as* $x \to \infty, f(x) \to$ ∞

as $x \to -\infty, f(x) \to$ $-\infty$

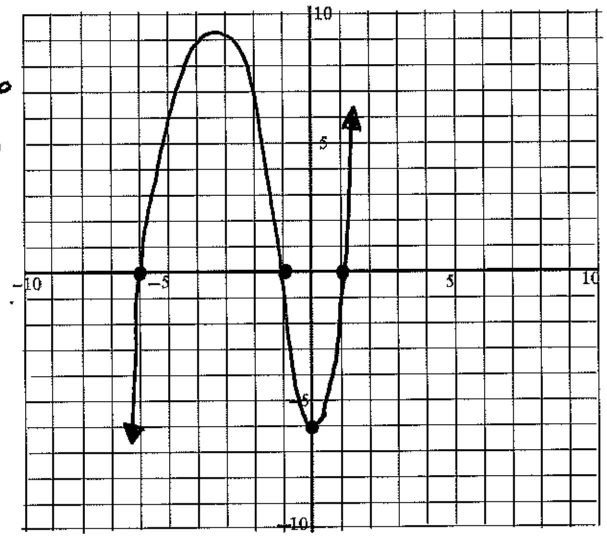
Zero(s): $-6, -1, 1$

Relative Max: $(-4.08, 30.04)$

Relative Min: $(0.08, -6.04)$

Intervals Increasing: $(-\infty, -4.08) \cup (0.08, \infty)$

Intervals Decreasing: $(-4.08, 0.08)$



$\quad -\quad +\quad +\quad +\quad +\quad -\quad +\quad +$

24.) Given $f(x) = x^7 + 4x^6 - 2x^5 + x^4 - 2x^3 - 2x^2 - 3x + 5$ complete the table below with the possible combinations of positive real zeros, negative real zeros, and complex zeros.

Positive Real Zeros	Negative Real Zeros	Complex Zeros	Total Zeros
4	3	0	7
2	1	4	7
0	1	6	7

25.) A florist delivers flowers to anywhere in town. d is the distance from the delivery address to the florist shop in miles. The cost to deliver flowers, based on the distance d, is given by $(d) = 0.04d^3 - 0.65d^2 + 3.5d + 9$. Evaluate $C(d)$ for $d = 6$ and $= 11$, and describe what the values of the function represent.

$C(6) = 0.04(6)^3 - 0.65(6)^2 + 3.5(6) + 9$	$C(11) = 0.04(11)^3 - 0.65(11)^2 + 3.5(11) + 9$
$C(6) = 15.24$	$C(11) = 22.09$
$C(6) =$ \$15.24.	$C(11) =$ \$22.09
THE COST FOR A 6 MILE DELIVERY DISTANCE IS \$15.24	THE COST FOR A 11 MILE DELIVERY DISTANCE IS \$22.09

26.) A jewelry box has a length that is 2 inches longer than the width and a height that is 1 inch smaller than the width. The volume of the box is 140 cubic inches. What is the width of the jewelry box? (Hint: $V = lwh$)

Length: $x + 2$

Width: x

Height: $x - 1$

- $140 = x(x+2)(x-1)$
- $140 = (x^2+2x)(x-1)$
- $140 = x^3 - x^2 + 2x^2 - 2x$
- $140 = x^3 + x^2 - 2x$
- $0 = x^3 + x^2 - 2x - 140$
- $x = 5$ (USE CALCULATOR TO FIND ZERO)

WIDTH = 5 INCHES

27.) The profit P (in millions of dollars) for a T-shirt manufacturer can be modeled by $P(x) = -x^3 + 4x^2 + x$ where x is the number of T-shirts produced (in millions). Currently, the company produced 4 million T-shirts and makes a profit of $\$4,000,000$. What lesser number of T-shirts could the company produce and still make the same profit?

- $4 = -x^3 + 4x^2 + x$
- $0 = -x^3 + 4x^2 + x - 4$
- $0 = (-x^3 + 4x^2) + (x - 4)$
- $0 = -x^2(x-4) + 1(x-4)$
- $0 = (x-4)(-x^2+1)$
- $x - 4 = 0 \qquad -x^2 + 1 = 0$

$x = 4$

$-x^2 = -1$

$x^2 = 1$

$x = \pm 1$

THEY CAN SELL 1 MILLION T-SHIRTS AND MAKE THE SAME AMOUNT OF PROFIT

28.) During soccer practice, Jill decided to see how high she can kick a soccer ball straight-up in the air. Her teammate, Jordan, who loves math, calculated that the height, in feet, of the soccer ball can be modeled by the equation $h(t) = -3t^2 + 24t + 3$, for t seconds.

a.) What was the initial height of the soccer ball when Jill's foot struck the ball? $\boxed{3\text{ FT}}$

- $h(0) = -3(0)^2 + 24(0) + 3$
- $h(0) = 3$

b.) When will the soccer ball reach its maximum height? $\boxed{4\text{ SECONDS}}$

- $t = \frac{-B}{2A} = \frac{-24}{2(-3)} = \frac{-24}{-6} = 4$

c.) What will be the maximum height of the soccer ball? $\boxed{51\text{ FT}}$

- $h(4) = -3(4)^2 + 24(4) + 3$
- $h(4) = -3(16) + 96 + 3$
- $h(4) = 51$

d.) When will the soccer ball hit the ground? $\boxed{8.12\text{ SECONDS}}$

- $0 = -3t^2 + 24t + 3$
- $0 = -3(t^2 - 8t - 1)$
- $0 = t^2 - 8t - 1$
- $1 = t^2 - 8t$
- $1 + \underline{16} = t^2 - 8t + \underline{16}$

$\left(\frac{-8}{2}\right)^2$

$(-4)^2$

16

- $17 = (t-4)^2$
- $\pm\sqrt{17} = t - 4$
- $4 \pm \sqrt{17} = t$

$t = 4 + \sqrt{17} = 8.12$

$t = 4 - \sqrt{17} = -0.12$

e.) When would the soccer ball reach a height of 30 feet? $\boxed{1.35\text{ SECONDS AND }6.65\text{ SECONDS}}$

- $30 = -3t^2 + 24t + 3$
- $0 = -3t^2 + 24t - 27$
- $0 = -3(t^2 - 8t + 9)$
- $0 = t^2 - 8t + 9$
- $t = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(9)}}{2(1)}$
- $t = \frac{8 \pm \sqrt{64 - 36}}{2}$
- $t = \frac{8 \pm \sqrt{28}}{2}$
- $t = \frac{8 \pm 2\sqrt{7}}{2}$
- $t = 4 \pm \sqrt{7}$
- $t = 6.65$ $t = 1.35$