Algebra 2
Semester 2
Finals Review #2

Name: KEY

Date: ____________ Per: ______

1.) Bacteria in a culture are growing exponentially with time, as shown in the table below.

a.) Fill in the table.

Bacteria Growth	
Day	**Bacteria**
0	100
1	200
2	400
3	800
4	1600
5	3200
6	6400

b.) Write an explicit rule that models the scenario above.

$$A_n = (200)(2)^{n-1}$$

c.) How much bacteria would be present by the 14th day?

$$A_{14} = (200)(2)^{14-1}$$

$$A_{14} = (200)(2)^{13}$$

$$A_{14} = 1638400$$

d.) How long would it take for this particular bacteria to reach 1,000,000?

$$1000000 = (200)(2)^{n-1}$$

$$5000 = (2)^{n-1}$$

$$\log_2 5000 = n-1$$

$$n = (\log_2 5000) + 1$$

$$n = 13.29$$

e.) What was the total bacteria growth on the 18th day?

$$S_{18} = \frac{18}{2}(200 + 2621440)$$

$$A_{18} = (200)(2)^{18-1}$$

$$A_{18} = 26214400$$

$$S_{18} = 235931400 + \underbrace{100}_{\text{INITIAL}}$$

$$S_{18} = 235{,}931{,}500$$

2.) A rumor is spreading through the halls of Dundee-Crown that Honors Algebra 2 is the Best Subject Ever! The table shows the number of people $P(t)$ who have heard the rumor t minutes after the rumor was started.

t	0	1	2	3	4
$P(t)$	2	12	22	32	42

+10 +10 +10 +10

a.) Write an explicit rule that models the scenario above.

$A_N = 12 + (N-1)(10)$

$A_N = 12 + 10N - 10$

$A_N = 10N + 2$

b.) How many people would hear the rumor by the 14th minute?

$A_{14} = 10(14) + 2$

$A_{14} = 140 + 2$

$A_{14} = 142$

c.) How many hours would it take for this rumor, we'll call it the "truth", to spread to 9,992?

$9992 = 10N + 2$

$9990 = 10N$

$N = 999$ MINS

$\frac{999 \text{ MINS}}{60 \text{ MINS}} = 16.65$ HOURS

d.) How many people, all together, would know the truth, I mean rumor, by the end of the 40 minutes?

$S_{40} = \frac{40}{2}(12 + 402)$

$S_{40} = 20(414)$

$S_{40} = 8280$

$A_{40} = 10(40) + 2$

$A_{40} = 400 + 2$

$A_{40} = 402$

3.) A savings account pays 4% interest compounded continuously. You deposit \$6000 into the account. If you neither add nor withdraw any money from the account, how much money (to the nearest cent) will you have in 5 years?

$A = Pe^{rt}$

$A = 6000e^{(.04)(5)}$

$A = \$7328.42$

4.) In a bacterial culture, the number B of bacteria is modeled by the equation $B = 15{,}000e^{0.27t}$, where t represents the number of hours since noon.

a.) How many bacteria (nearest whole number) will be present at 5:00 p.m.?

$B = 15000\,e^{0.27(5)}$

$B = 57861.38$

57861 BACTERIA

b.) How many hours (correct to three decimal places) will it take for there to be 120,000 bacteria?

$120000 = 15000\,e^{.27t}$

$8 = e^{.27t}$

$\ln 8 = .27t$

$t = \dfrac{\ln 8}{.27}$

$t = 7.702$

7.702 HOURS

5.) Graph the exponential function and find all the information listed below.

$f(x) = (2)\left(\frac{1}{2}\right)^{x+1} - 6$

x	y
0	2
1	1

Parent Function: $y = (1/2)^x$

Growth / Decay: DECAY

Asymptote: $y = -6$

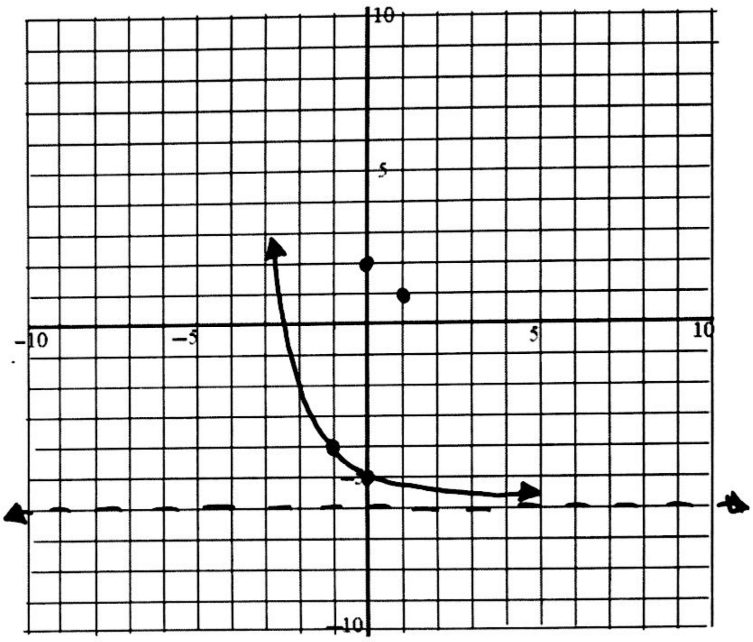
Domain: $(-\infty, \infty)$

Range: $(-6, \infty)$

End Behavior:

As $x \to \infty$, $f(x) = -6$

As $x \to -\infty$, $f(x) = \infty$



Describe the transformation(s):

HORIZONTAL SHIFT LEFT 1

VERTICAL SHIFT DOWN 6

VERTICAL STRETCH OF 2

Find the zero(s) algebraically.

$0 = (2)(1/2)^{x+1} - 6$

$6 = (2)(1/2)^{x+1}$

$3 = (1/2)^{x+1}$

$\log_{1/2} 3 = x + 1$

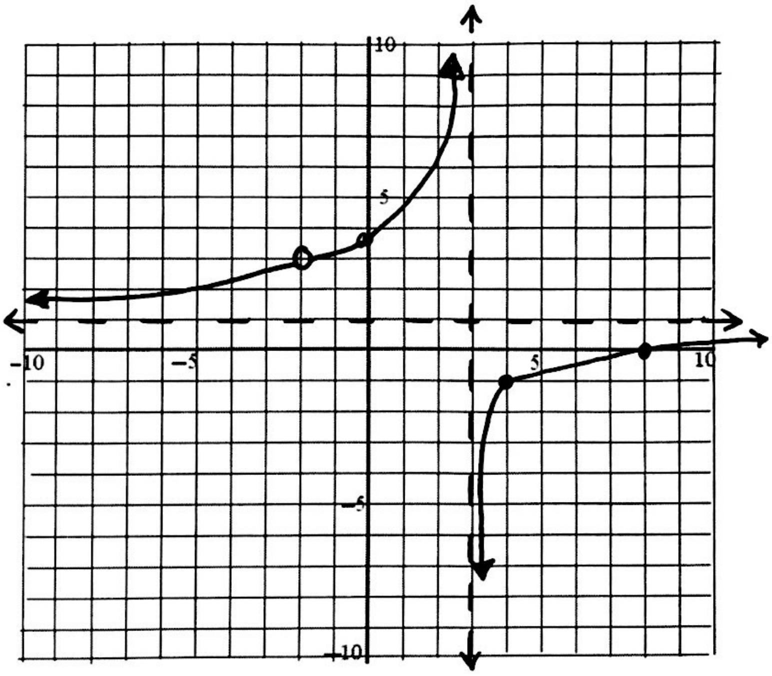
$x = (\log_{1/2} 3) - 1$

$x = -2.5850$

Zero(s): $x = -2.5850$

6.) Graph the rational function and find all the information listed below.

$f(x) = \frac{x^2 - 6x - 16}{x^2 - x - 6} = \frac{(x-8)\cancel{(x+2)}}{(x-3)\cancel{(x+2)}} = \frac{(x-8)}{(x-3)}$ HOLE



y-intercept: $(0, 8/3)$ or $(0, 2.67)$

Hole(s): $x = -2$ $(-2, 2)$

Zero(s): $x = 8$

Vertical Asymptote(s): $x = 3$

Horizontal Asymptote(s): $y = 1$

Domain: $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

Range: $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

End Behavior:

$as\ x \to \infty, f(x) \to$ 1

$as\ x \to -\infty, f(x) \to$ 1

7.) Perform Indicated Operation

a.) $\frac{x+7}{x^2-9} \div \frac{x^2+9x+14}{3x^2-9x}$

$= \frac{\cancel{(x+7)}}{(x+3)\cancel{(x-3)}} \cdot \frac{3x\cancel{(x-3)}}{\cancel{(x+7)}(x+2)}$

$= \frac{3x}{(x+3)(x+2)}$

b.) $\frac{(x-1)}{(x-1)} \frac{3x}{x-5} - \frac{2x+3}{x^2-6x+5}$ $(x-5)(x-1)$

$= \frac{3x^2 - 3x}{(x-5)(x-1)} - \frac{(2x+3)}{(x-5)(x-1)}$

$= \frac{3x^2 - 5x - 3}{(x-5)(x-1)}$

8.) Convert to degrees: $\frac{4\pi}{3}$

$= \frac{4\pi}{3} \cdot \frac{180}{\pi}$

$= 240°$

9.) Covert to radians: 330°

$= 330° \cdot \frac{\pi}{180}$

$= \frac{11\pi}{6}$

Solve each of the following exponential equations.

10.) $\left(\frac{1}{81}\right)^{3-x} = 243^{2x+6}$

$3^{-4(3-x)} = 3^{5(2x+6)}$

$-12 + 4x = 10x + 30$

$-12 = 6x + 30$

$-42 = 6x$

$\boxed{-7 = x}$

11.) $\log_4(x-2) + \log_4(x+4) = 2$

$\log_4 (x-2)(x+4) = 2$

$4^2 = (x-2)(x+4)$

$16 = x^2 + 2x - 8$

$0 = x^2 + 2x - 24$

$0 = (x+6)(x-4)$

$x = -6$ $\boxed{x = 4}$

EXTRANEOUS

12.) $\log_4(3x+14) - \log_4 5 = \log_4 2x$

$\log_4\left(\frac{3x+14}{5}\right) = \log_4 2x$

$\frac{3x+14}{5} = \frac{2x}{1}$

$10x = 3x + 14$

$7x = 14$

$\boxed{x = 2}$

13.) $\log_8(m-3) + \log_8(m+4) = 1$

$\log_8 (M-3)(M+4) = 1$

$8^1 = (M-3)(M+4)$

$8 = M^2 + M - 12$

$0 = M^2 + M - 20$

$0 = (M+5)(M-4)$

$M = -5$ $\boxed{M = 4}$

EXTRANEOUS

14.) $16e^{2x-6} = 64$

$e^{2x-6} = 4$

$\ln 4 = 2x - 6$

$2x = (\ln 4) + 6$

$x = \frac{(\ln 4) + 6}{2}$

$\boxed{x = 3.6931}$

15.) $4^{n+2} = 14.5$

$\log_4 14.5 = n + 2$

$n = (\log_4 14.5) - 2$

$\boxed{n = -0.0710}$

16.) Solve. Remember to check for extraneous solutions.

a.) $\left[\frac{4}{x^2-8x+12} = \frac{x}{x-2} + \frac{1}{x-6}\right](x-6)(x-2)$

$(x-6)(x-2)$

$4 = x(x-6) + 1(x-2)$

$4 = x^2 - 6x + x - 2$

$0 = x^2 - 5x - 6$

$0 = (x-6)(x+1)$

$x = 6$ $\boxed{x = -1}$

EXTRANEOUS

b.) $\left[\frac{5}{n} - \frac{6}{n^3-2n^2} = \frac{n^2+5n-6}{n^3-2n^2}\right] n^2(n-2)$

$n^2(n-2)$ $n^2(n-2)$

$5(n-2)(n) - 6 = n^2 + 5n - 6$

$5n^2 - 10n - 6 = n^2 + 5n - 6$

$4n^2 - 15n = 0$

$n(4n - 15) = 0$

$n = 0$ $4n - 15 = 0$

EXTRANEOUS $\boxed{n = 15/4}$

17.) Suppose a new club is being formed and they need to select a President, a Vice-President, and a Treasurer from a group of 20 individuals. How many different arrangements can be formed?

$_{20}P_3 = 6840$

18.) Suppose a new club is being formed and they need to select four individuals from a group of 20 individuals to run the club. How many different arrangements can be formed?

$_{20}C_4 = 4845$

19.) Verify the following identity.

a.) $\frac{sec\theta\ sin\theta}{tan\theta} - 1 = 0$

$\frac{\frac{1}{\cos\theta} \cdot \sin\theta}{\tan\theta} - 1 = 0$

$\frac{\frac{\sin\theta}{\cos\theta}}{\tan\theta} - 1 = 0$

$\frac{\tan\theta}{\tan\theta} - 1 = 0$

$1 - 1 = 0$

$0 = 0$

b.) $\frac{tan\theta}{1-cos^2\theta} = sec\theta csc\theta$

$\frac{\tan\theta}{\sin^2\theta} =$

$\tan\theta \div \sin^2\theta =$

$\frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sin^2\theta} =$

$\frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} =$

$\sec\theta \cdot \csc\theta =$

Evaluate each.

20.) $\sum_{n=1}^{40}[6+(n-1)2]$

$S_N = \frac{N}{2}(A_1 + A_N)$

$S_{40} = \frac{40}{2}(6 + 84)$

$S_{40} = 20(90)$

$\boxed{S_{40} = 1800}$

21.) $\sum_{n=1}^{10} 4 \cdot 2^{n-1}$

$S_N = \frac{A_1(1-R^N)}{(1-R)}$

$S_{10} = \frac{4(1-2^{10})}{(1-2)}$

$\boxed{S_{10} = 4092}$

22.) $\sum_{n=3}^{9}\left(\frac{2}{3}\right)\left(3^{n-1}\right)$

$S_N = \frac{A_1(1-R^N)}{(1-R)}$

$S_7 = \frac{(2/3)(1-3^7)}{(1-3)}$

$\boxed{S_7 = 728.67}$

23.) $\sum_{n=1}^{\infty}(8)\left(\frac{3}{4}\right)^{n-1}$

$S_\infty = \frac{A_1}{(1-R)}$

$S_\infty = \frac{8}{(1-3/4)}$

$\boxed{S_\infty = 32}$