Algebra 2
Final Exam Review – Semester 2

Name: KEY
Date: _____ Period: ____

1.) **Bacteria in a culture are growing exponentially with time, as shown in the table below.**

a.) **Fill in the table.**

Bacteria Growth	
Day	**Bacteria**
0	100
1	200
2	400
3	800
4	1600
5	3200
6	6400

b.) **Write an explicit rule that models the scenario above.**

$A_N = A_1 \cdot R^{N-1}$

$A_N = (200)(2)^{N-1}$

c.) **How much bacteria would be present by the 14th day?**

$A_N = (200)(2)^{N-1}$

$A_{14} = (200)(2)^{14-1}$

$A_{14} = (200)(2)^{13}$

1,638,400 CELLS

2.) **In a bacterial culture, the number B of bacteria is modeled by the equation $B = 15,000e^{0.27t}$, where t represents the number of hours since noon.**

a.) **How many bacteria (nearest whole number) will be present at 5:00 p.m.?**

$B = 15,000\,e^{0.27(5)}$

$B = 57861.383$

57,861 CELLS

b.) **How many hours (correct to three decimal places) will it take for there to be 120,000 bacteria?**

$t = ?$

$\frac{120,000}{15,000} = \frac{15,000\,e^{.27t}}{15,000}$

$8 = e^{.27t}$

$\frac{\ln 8}{.27} = \frac{.27t}{.27}$

$t = 7.702$

7.702 Hours

3.) A rumor is spreading through the halls of Dundee-Crown that Algebra 2 is the Best Subject Ever! The table shows the number of people $P(t)$ who have heard the rumor t minutes after the rumor was started.

t	0	1	2	3	4
$P(t)$	2	12	22	32	42

a.) Write an explicit rule that models the scenario above.

$A_N = A_1 + (N-1)(D)$

$A_N = 12 + (N-1)(10)$

$A_N = 12 + 10N - 10$

$A_N = 10N + 2$

b.) How many people would hear the rumor by the 14th minute?

$A_N = 10N + 2$

$A_{14} = 10(14) + 2$

$A_{14} = 142$

142 PEOPLE

c.) How many hours would it take for this rumor, we'll call it the "truth", to spread to 9,992 students?

$A_N = 10N + 2$

$9992 = 10N + 2$

$9990 = 10N$

$N = 999$

999 MINS = 16.65 HOURS

d.) How many people, all together, would know the truth, I mean rumor, by the end of the 40 minutes?

$S_N = \frac{N}{2}(A_1 + A_N)$ SUM

$S_{40} = \frac{40}{2}(12 + 402)$

$A_N = 10N + 2$

$A_{40} = 10(40) + 2$

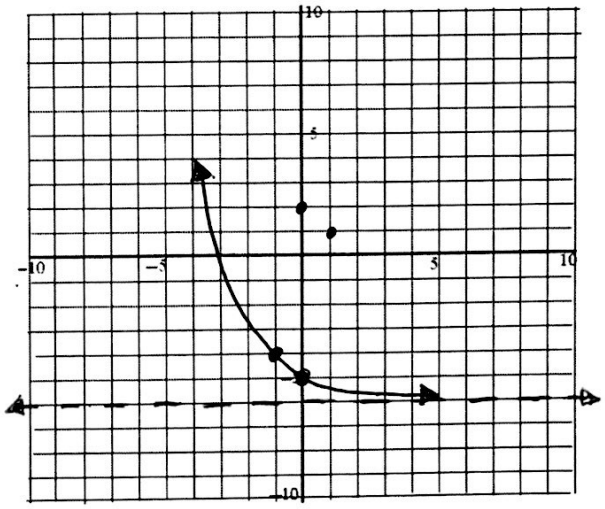
$A_{40} = 402$

8280 PEOPLE

4.) Graph the exponential function and find all the information listed below.

$f(x) = (2)\left(\frac{1}{2}\right)^{x+1} - 6$

x	y
0	2
1	1



Parent Function: $y = \left(\frac{1}{2}\right)^x$

Growth / Decay: DECAY

Asymptote: $y = -6$

Domain: $(-\infty, \infty)$

Range: $(-6, \infty)$

End Behavior:

As $x \to$ ∞, $f(x) =$ -6

As $x \to$ $-\infty$, $f(x) =$ ∞

Describe the transformation(s):

VERTICAL STRETCH OF 2

HORIZONTAL SHIFT OF LEFT 1

VERTICAL SHIFT OF DOWN 6

Find the zero(s) algebraically. $(\#, 0)$

$0 = (2)(1/2)^{x+1} - 6$

$6 = (2)(1/2)^{x+1}$

$3 = (1/2)^{x+1}$

$\log_{(1/2)} 3 = x + 1$

$x = \left(\log_{(1/2)} 3\right) - 1$

$x = -2.5850$

5.) **You are looking to invest $6000 into an account for your retirement. If you neither add nor withdraw any money from the account, which of the following options would be the best account to invest in after 40 years?**

-Option 1: Offers 3.8% interest compounded yearly

-Option 2: Offers 3.2% interest compounded continuously

-Option 3: Offers 3.5% compounded monthly

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 6000(1 + .038)^{1(40)}$

$A = 6000(1 + .038)^{40}$

$A = 26,671.37$

OPTION 1

Option 1 total: $26,671.37

$A = Pe^{rt}$

$A = 6000e^{.032(40)}$

$A = 21579.84$

OPTION 2

Option 2 total: $21,579.84

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 6000\left(1 + \frac{.035}{12}\right)^{(12)(40)}$

$A = 24,281.67$

OPTION 3

Option 3 total: $24,281.67

Best Investment: OPTION 1

6.) **Perform Indicated Operation**

a.) $\frac{x+7}{x^2-9} \div \frac{x^2+9x+14}{3x^2-9x}$

$= \frac{(x+7)}{(x+3)(x-3)} \cdot \frac{3x(x-3)}{(x+7)(x+2)}$

$= \frac{3x}{(x+3)(x+2)}$

b.) $\frac{(x-1)}{(x-1)}\frac{3x}{x-5} - \frac{2x+3}{x^2-6x+5}$

$(x-5)(x-1)$

$= \frac{3x^2-3x}{(x-5)(x-1)} - \frac{(2x+3)}{(x-5)(x-1)}$

$= \frac{3x^2-5x-3}{(x-5)(x-1)}$

7.) Solve. Remember to check for extraneous solutions.

a.) $\left[\frac{4}{x^2-8x+12} = \frac{x}{x-2} + \frac{1}{x-6}\right](x-6)(x-2)$

$(x-6)(x-2)$

$4 = x(x-6) + (x-2)$

$4 = x^2 - 6x + x - 2$

$0 = x^2 - 5x - 6$

$0 = (x-6)(x+1)$

$x = 6$ $\boxed{x = -1}$

EXTRANEOUS

b.) $\left[\frac{5}{n} - \frac{6}{n^3-2n^2} = \frac{n^2+5n-6}{n^3-2n^2}\right] n^2(n-2)$

$n^2(n-2)$ $n^2(n-2)$

$5n(n-2) - 6 = n^2 + 5n - 6$

$5n^2 - 10n - 6 = n^2 + 5n - 6$

$4n^2 - 15n = 0$

$n(4n-15) = 0$

$n = 0$ $4n - 15 = 0$

EXTRANEOUS $4n = 15$

$\boxed{n = 15/4}$

8.) Suppose a new club is being formed and they need to select a President, a Vice-President, and a Treasurer from a group of 20 individuals. How many different arrangements can be formed?

* ORDER MATTERS

$20\ P_3$

$\boxed{6840}$

9.) Suppose a new club is being formed and they need to select four individuals from a group of 20 individuals to run the club. How many different arrangements can be formed?

$20\ C_4$

$\boxed{4845}$

10.) The manager of a candy store in downtown Carpentersville is interested in assessing customer opinions about adding new candy on its shelves. The manager is planning to conduct a sample survey of the customer population.

Which of the listed methods of sample selection would be most efficient at reducing bias?

A. Post the survey on the store Web site, and use the first 30 surveys that are submitted.

B. Randomly select 30 customer surveys submitted throughout the week.

OR

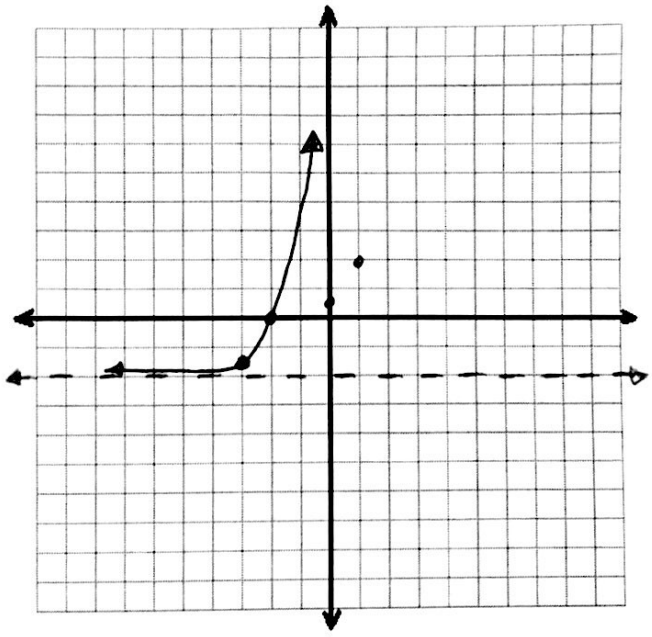
C. Randomly select one day of the week, and then select the first 30 customers who enter the store after 12pm.

D. Randomly select one day of the week, and then select the first 30 customers who enter the store that day.

Graph the exponential function and find all the information listed below.

11.) $f(x) = \frac{1}{2}(4)^{x+3} - 2$

x	y
0	1/2
1	2



Parent Function: $y = 4^x$

Growth / Decay: GROWTH

Asymptote: $y = -2$

Domain: $(-\infty, \infty)$

Range: $(-2, \infty)$

End Behavior:

$As\ x \to -\infty, f(x) \to$ -2

$As\ x \to \infty, f(x) \to$ ∞

Describe the transformation(s), in the order they are made:

HORIZONTAL SHIFT OF LEFT 3, VERTICAL SHRINK OF 1/2,

VERTICAL SHIFT OF DOWN 2

12.) Mr. Falinski has decided to purchase two pet bunnies for each of his two children. His two daughters each receive a boy bunny and a girl bunny. Not knowing the severe risk of owning multiple bunnies, let alone keeping them in the same cage, Mr. Falinski is now the proud owner of a rapidly increasing population of bunnies. Mr. Falinski was able to calculate (go figure) that the bunnies were doubling every year.

a. Fill in the chart. (Please round appropriately.)

# of Years	0	1	2	3
# of Bunnies	2	4	8	16

b. State the common difference or the common ratio. (Include the appropriate symbol.)

$R = 2$

c. Write an explicit rule that models the scenario above.

$A_N = A_1 \cdot R^{N-1}$

$A_N = (4)(2)^{N-1}$

d. How many bunnies will Mr. Falinski have by the 8th year?

$A_8 = (4)(2)^{8-1}$

$A_8 = (4)(2)^7$

$A_8 = 512$

Bunnies: 512

Use the two way frequency table to answer the following questions.

	Workouts 1-2 Times Per Week	Workouts 3 Times a Week	3+ Times Per Week	Total
Male	62	37	55	154
Female	73	89	33	195
Total	135	126	88	349

13.)Find the probability that a person who is randomly selected...

a.) is a male?

$\frac{154}{349} = .44 = 44\%$

b.) is a female?

$\frac{195}{349} = .56 = 56\%$

c.) exercises 1-2 times per week?

$\frac{135}{349} = .39 = 39\%$

d.) exercises 3+ times per week?

$\frac{88}{349} = .25 = 25\%$

e.) exercises 0-2 times per week?

f.) is a male that exercises 0 times/week? $\frac{0}{154} = 0\%$

Solve each equation. If necessary, round answers to 4 decimal places.

14.) $\log_3(5m+1) = \log_3(m^2-49)$

$5M+1 = M^2-49$

$0 = M^2-5M-50$

$0 = (M-10)(M+5)$

$\boxed{M=10}$ $M=-5$ EXTRANEOUS

15.) $(2)^{x-1} = 32^{x+3}$

$2^{x-1} = 2^{5(x+3)}$

$x-1 = 5x+15$

$-1 = 4x+15$

$-16 = 4x$

$\boxed{x=-4}$

16.) $3^{2x^2} \cdot 3^{5x} = 27$

$3^{2x^2+5x} = 27$

$3^{2x^2+5x} = 3^3$

$2x^2+5x = 3$

$2x^2+5x-3 = 0$

$(x+3)(2x-1) = 0$

$x+3=0$ $2x-1=0$

$\boxed{x=-3}$ $\boxed{x=1/2}$

17.) $2+7\log_{11}(3m-8) = 16$

$7\log_{11}(3M-8) = 14$

$\log_{11}(3M-8) = 2$

$11^2 = 3M-8$

$121 = 3M-8$

$129 = 3M$

$\boxed{M=43}$

18.) Mr. Tollberg is looking to invest $1200 in a local bank for his retirement. He is thinking of keeping the money in this bank for 25 years. The bank has offered him multiple options for investing.

Option 1 is an account the offers a 3.1% annual interest rate compounded yearly.
Option 2 is an account that offers a 2.8% annual interest rate compounded monthly.
Option 3 is an account that offers a 2.5% annual interest rate compounded continuously.

a. If Mr. Tollberg picked option 1, how much money would he have after 25 years?

$A = P(1+r)^t$

$A = 1200(1+.031)^{25}$

$A = \$2574.23$

$\$2574.23$

b. If Mr. Tollberg picked option 2, how much money would he have after 25 years?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 1200\left(1+\frac{.028}{12}\right)^{(12)(25)}$

$A = \$2414.53$

$\$2414.53$

c. If Mr. Tollberg picked option 3, how much money would he have after 25 years?

$A = Pe^{rt}$

$A = 1200e^{(.025)(25)}$

$A = \$2241.90$

$\$2241.90$

19.) Maddie was so excited to find a random bag on the floor that contained 8 red marbles, 4 green marbles, 3 blue marbles, and 5 yellow marbles.

a.) What is the probability that Maddie reaches in the bag and selects a green marble and then a yellow marble if she doesn't replace the first marble?

$\frac{4}{20} \cdot \frac{5}{19} = \frac{20}{380} = \frac{1}{19} = 0.053 = 5.3\%$

b.) What is the probability that Maddie reaches in the bag and selects a red marble and then another red marble if she replaces the first marble in the bag?

$\frac{8}{20} \cdot \frac{8}{20} = \frac{16}{400} = \frac{1}{25} = 0.04 = 4\%$

c.) What is the probability that Maddie reaches in a selects a blue marble, then another blue marble, and then a red marble if she doesn't replace the marbles?

$\frac{3}{20} \cdot \frac{8}{19} \cdot \frac{2}{18} = \frac{48}{6840} = \frac{2}{285} = 0.007 = 0.7\%$

20.) A club elects a vice-president, treasurer and a secretary. How many different sets of officers are possible if there are 15 members and any of them can be chosen for any positon? No person can hold more than one office.

$15 P_3 = 2730$

21.) How many different ways can a chairperson and an assistant chairperson be selected for a research project if there are seven scientists available?

$7 P_2 = 42$

22.) Use the following two way frequency table to answer the following questions.

	Spanish	French	German	Total
Boys	10	2	8	20
Girls	15	12	3	30
Total	25	14	11	50

Find the probability that a person who is randomly selected...

a.) took Spanish?

$\frac{25}{50} = \frac{1}{2} = 0.5 = 50\%$

b.) took French?

$\frac{14}{50} = \frac{7}{25} = .28 = 28\%$

c.) took German?

$\frac{11}{50} = .22 = 22\%$

d.) What is the probability that a randomly selected girl took French?

$\frac{12}{30} = \frac{2}{5} = 0.4 = 40\%$

e.) What is the probability that randomly selected boy took German?

$\frac{8}{20} = \frac{2}{5} = 0.4 = 40\%$

23.) Use the equation to answer the questions.

$y = 5 \sin 5\theta - 4$

Amplitude: 5

Period: $\frac{360}{5} = 72$

Vertical Shift: Down 4

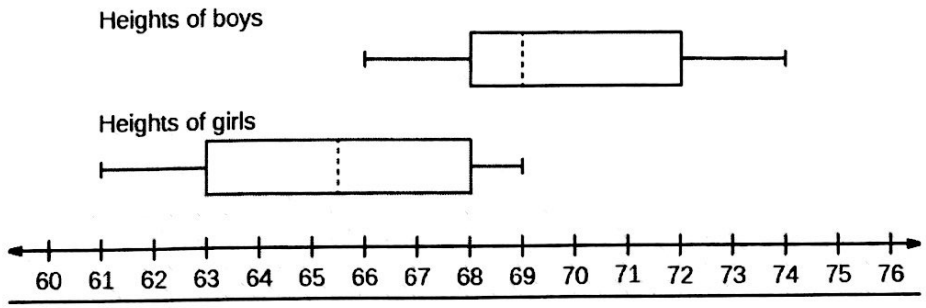
$y = 2 \cos 3\theta + 3$

Amplitude: 2

Period: $\frac{360}{3} = 120°$

Vertical Shift: up 3

24.) Use the box plot below to answer the following questions about the heights of boys and girls in inches.



a.) What was the maximum height of the boys? 74 in Girls? 69 in

b.) What percent of the girls are between 61 inches and 68 inches? 75%

c.) Based on the box plot, 50% of the boys are above what height? 69 in

d.) Based on the box plot, 75% of the girls are above what height? 63 in

25.) Determine the angle with the same reference angle as 215° in...

1.) Q1 35°

2.) Q2 145°

3.) Q3 215°

4.) Q4 325°

26.) Determine the angle with the same reference angle as 327° in...

1.) Q1 33°

2.) Q2 147°

3.) Q3 213°

4.) Q4 327°

27.) Find the exact value for the following values.....

a. cos 60° $1/2$

b. sin 225° $-\sqrt{2}/2$

c. tan 300° $-\sqrt{3}$

28.) What are the coordinates of B:

$(-1/2, \sqrt{3}/2)$