Algebra 2

Review – Roots, Radicals, and Rational Exponents

Name: KEY

Date: _____ Period: _____

Simplify.

1.) $\sqrt[4]{48x^8y^{14}}$

$\sqrt[4]{16x^8y^{12}} \cdot \sqrt[4]{3y^2}$

$\boxed{2x^2y^3\sqrt[4]{3y^2}}$

2.) $\sqrt[3]{8xy^7} \cdot \sqrt[3]{6x^6y^3}$

$\sqrt[3]{48x^7y^{10}}$

$\sqrt[3]{8x^6y^9} \cdot \sqrt[3]{6xy}$

$\boxed{2x^2y^3\sqrt[3]{6xy}}$

3.) $(\sqrt[3]{9a^8b^6})(\sqrt[3]{6a^2b})$

$\sqrt[3]{54A^{10}B^7}$

$\sqrt[3]{27A^9B^6} \cdot \sqrt[3]{2AB}$

$\boxed{3A^3B^2\sqrt[3]{2AB}}$

4.) $3\sqrt[3]{81} + 5\sqrt[3]{24}$

$3\sqrt[3]{27} \cdot \sqrt[3]{3} + 5\sqrt[3]{8} \cdot \sqrt[3]{3}$

$3 \cdot 3 \cdot \sqrt[3]{3} + 5 \cdot 2 \cdot \sqrt[3]{3}$

$9\sqrt[3]{3} + 10\sqrt[3]{3}$

$\boxed{19\sqrt[3]{3}}$

5.) $\dfrac{\sqrt[3]{81x^5y^8}}{\sqrt[3]{3x^2y}}$

$\sqrt[3]{27x^3y^7}$

$\sqrt[3]{27x^3y^6} \cdot \sqrt[3]{y}$

$\boxed{3xy^2\sqrt[3]{y}}$

6.) $\sqrt[4]{32} - 3\sqrt[4]{162} + 2\sqrt[4]{48}$

$\sqrt[4]{16} \cdot \sqrt[4]{2} - 3\sqrt[4]{81} \cdot \sqrt[4]{2} + 2\sqrt[4]{16}\sqrt[4]{3}$

$2\sqrt[4]{2} - 3 \cdot 3 \cdot \sqrt[4]{2} + 2 \cdot 2 \cdot \sqrt[4]{3}$

$2\sqrt[4]{2} - 9\sqrt[4]{2} + 4\sqrt[4]{3}$

$\boxed{-7\sqrt[4]{2} + 4\sqrt[4]{3}}$

7.) Convert between forms in the chart below.

Radical Form	Rational Exponent Form
$\sqrt[3]{5}$	$5^{1/3}$
$(\sqrt[5]{2x})^3$	$(2x)^{\frac{3}{5}}$
$(\sqrt[5]{(x+2)})^4$	$(x+2)^{4/5}$
$(\sqrt[3]{4})^4$	$4^{\frac{4}{3}}$

8.) Write the radical function that results from applying the following transformation to the graph of $y = \sqrt{x}$.

- A translation of 4 units to the left and 1 unit down.
- A vertical shrink of $\frac{1}{3}$.

$y = \frac{1}{3}\sqrt{x+4} - 1$

9.) Write the radical function that results from applying the following transformation to the graph of $y = \sqrt[3]{x}$.

- A translation of 2 units to the right and 3 unit up.
- A vertical stretch of 2.

$y = 2\sqrt[3]{x-2} + 3$

Graph the following radical functions and state the key information.

10.) $y = \frac{1}{2}\sqrt{x+4} - 1$

Parent Function: $y = \sqrt{x}$

Domain: $[-4, \infty)$

Range: $[-1, \infty)$

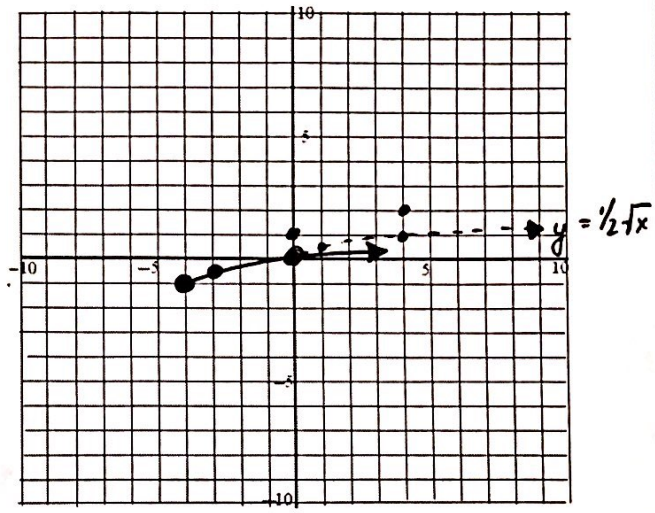
End Behavior:

as $x \to \infty, f(x) \to$ ∞

as $x \to -\infty, f(x) \to$ —

Zeros: 0 (WHERE GRAPH CROSSES X-AXIS)

x	y
0	0
1	1/2
4	2



Describe the shifts from the parent function.

VERTICAL SHRINK OF 1/2, HORIZONTAL SHIFT OF LEFT 4, VERTICAL SHIFT OF DOWN 1

11.) $y = 2\sqrt[3]{x-2} + 3$

Parent Function: $y = \sqrt[3]{x}$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

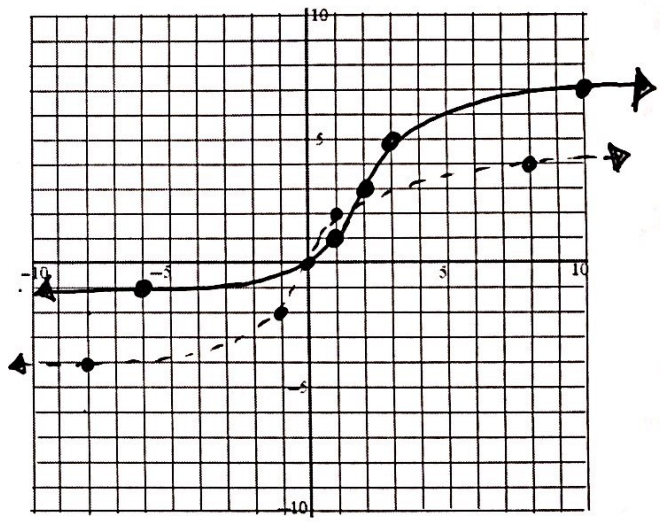
End Behavior:

as $x \to \infty, f(x) \to$ ∞

as $x \to -\infty, f(x) \to$ $-\infty$

Zeros: 0

x	y
-8	-4
-1	-2
0	0
1	2
8	4



Describe the shifts from the parent function.

VERTICAL STRETCH OF 2, HORIZONTAL SHIFT OF RIGHT 2, VERTICAL SHIFT OF UP 3

Solve. Remember to check for extraneous solutions.

12.) $\left(\sqrt{3x+10}\right)^2 = \left(2 + \sqrt{3x-4}\right)^2$

$3x + 10 = 4 + 2\sqrt{3x-4} + 2\sqrt{3x-4} + 3x - 4$

$3x + 10 = 4\sqrt{3x-4} + 3x$

$(10)^2 = (4\sqrt{3x-4})^2$

$100 = 16(3x-4)$

$100 = 48x - 64$

$164 = 48x$

$x = \frac{164}{48}$

$\boxed{x = 41/12}$

13.) $3(9x-5)^{\frac{3}{2}} - 6 = 18$

$3(9x-5)^{3/2} = 24$

$\left[(9x-5)^{3/2}\right]^{2/3} = (8)^{2/3}$

$9x - 5 = (\sqrt[3]{8})^2$

$9x - 5 = (2)^2$

$9x - 5 = 4$

$9x = 9$

$\boxed{x = 1}$

14.) $\left[\sqrt[4]{4x+5}\right]^4 = \left[-3\right]^4$

$4x+5 = 81$

$4x = 76$

$x = 19$

$\boxed{\text{EXTRANEOUS}}$

$\sqrt[4]{4(19)+5} = -3$

$\sqrt[4]{76+5} = -3$

$\sqrt[4]{81} = -3$

$3 \neq -3$

15.) $4(2x+7)^{\frac{1}{2}} - 11 = 25$

$4(2x+7)^{1/2} = 36$

$\left[(2x+7)^{1/2}\right]^2 = \left[9\right]^2$

$2x+7 = 81$

$2x = 74$

$\boxed{x = 37}$

16.) $(x-4)^2 = (\sqrt{2x})^2$

$x^2 - 4x - 4x + 16 = 2x$

$x^2 - 8x + 16 = 2x$

$x^2 - 10x + 16 = 0$

$(x-8)(x-2) = 0$

$x - 8 = 0 \quad x - 2 = 0$

$\boxed{x = 8} \quad x = 2$

EXTRANEOUS

17.) $2(3x-4)^{\frac{2}{5}} - 5 = 3$

$2(3x-4)^{2/5} = 8$

$\left[(3x-4)^{2/5}\right]^{5/2} = \left[4\right]^{5/2}$

$3x - 4 = \left[4\right]^{5/2}$

$3x - 4 = \pm 32$

$3x - 4 = 32 \qquad 3x - 4 = -32$

$3x = 36 \qquad 3x = -28$

$\boxed{x = 12} \qquad \boxed{x = -28/3}$

Factor Completely.

-24

$-6 \quad 4$

-2

18.) $9m^2 - 25$

$\boxed{(3M-5)(3M+5)}$

19.) $3x^2 - 2x - 8$

$3x^2 - 6x + 4x - 8$

$(3x^2 - 6x) + (4x - 8)$

$3x(x-2) + 4(x-2)$

$\boxed{(x-2)(3x+4)}$

20.) $8w^3 - 27$

$2w \; 2w \; 2w \quad 3 \; 3 \; 3$

$\boxed{(2w-3)(4w^2 + 6w + 9)}$