Algebra 2 – 3316
9.6 – Applications of Sequences and Series
Show mathematical proof

Name: KEY
Date: ________________ Period: ______

1.) An auditorium has 20 seats on the first row, 24 seats on the second row, 28 seats on the third row, and so on and has 30 rows of seats. How many seats are in the theatre?

$A_N = A_1 + (N-1)(D)$

$A_{30} = 20 + (30-1)(4)$

$A_{30} = 20 + 116$

$A_{30} = 136$

$S_{30} = \frac{30}{2}(20 + 136)$

$S_{30} = 15(156)$

$S_{30} = 2340$

$\boxed{2340 \text{ SEATS}}$

2.) Suppose you go to work for a company that pays one penny on the first day, 2 cents on the second day, 4 cents on the third day and so on. If the daily wage keeps doubling, what will you total income be for working 31 days?

$S_N = \frac{A_1(1-R^N)}{1-R}$

$S_{31} = \frac{.01(1-2^{31})}{1-2}$

$S_{31} = 21474836.47$

$\boxed{\$21,474,836.47}$

3.) Logs are stacked in a pile with 24 logs on the bottom row and 15 on the top row. There are 10 rows in all with each row having one more log than the one above it. How many logs are in the stack?

$S_N = \frac{N}{2}(A_1 + A_N)$

$S_{10} = \frac{10}{2}(24 + 15)$

$S_{10} = 5(39)$

$S_{10} = 195$

$\boxed{195 \text{ LOGS}}$

4.) A ball is dropped from a height of 16 feet. Each time it drops, it rebounds 80% of the height from which it is falling. Find the total distance traveled in 15 bounces. 16, 12.8, 10.24

$S_N = \frac{A_1(1-R^N)}{(1-R)}$

$S_{15} = \frac{16(1-.8^{15})}{(1-.8)}$

$S_{15} = 77.19$

$\boxed{138.38 \text{ FEET}}$

5.) A company is offering a job with a salary of $30,000 for the first year and a 5% raise each year after that. If that 5% raise continues every year, find the amount of money you would earn in a 40-year career.

$S_N = \frac{A_1(1-R^N)}{1-R}$

$S_{40} = \frac{30000(1-1.05^{40})}{(1-1.05)}$

$S_{40} = 3623993.23$

$\boxed{\$3,623,993.23}$

6.) A culture of bacteria doubles in population every 2 hours. If there are 500 bacteria at the beginning, how many bacteria will there be after 24 hours? 500, 1000, 2000

$A_N = A_1 \cdot R^{N-1}$

$A_{12} = 500 \cdot 2^{12-1}$

$A_{12} = 1024000$

$\boxed{1,024,000 \text{ BACTERIA}}$

1, 6, 36, 216, ...

7.) A student with the swine flu comes to school and has the potential to infect 6 other students. Similarly those 6 students will each infect 6 more students. If this process is done 12 times, how many students will be infected?

$A_N = A_1 \cdot r^{N-1}$

$A_{12} = 1 \cdot 6^{12-1}$

$A_{12} = 362797056$

362,797,056 INFECTED STUDENTS JUST IN WEEK 12

8.) Bill Nye really wanted to create a cell that would regenerate. After two years of research he finally developed a cell that would regenerate each week. Below is the data he collected in the lab.

Week	Number of Cells
1	4
2	16
3	64
4	256
5	1,024
6	4096
7	16384
8	65536

A.) Find the number of cells for the remaining weeks.

B.) Write an explicit rule to represent this data.

$A_N = 4 \cdot 4^{N-1}$

C.) How many cells will there be in the 23rd week. Show mathematical proof.

$A_{23} = 4 \cdot 4^{23-1}$

$A_{23} = 7.0369 \times 10^{13}$

D.) How many cells would you need to start with to reach 5000 cells by week 6?

$5000 = A_1 \cdot 4^{6-1}$

$5000 = A_1 \cdot 1024$

$A_1 = 4.88$

$\boxed{5 \text{ CELLS}}$

9.) Bill Nye conducts a different experiment and gets the results below.

Let W_n represent the number of cells in the growth medium at the end of week n. Which of these statements are true about the explicit formula for W_n ? Select all that apply.

- ☐ $w_n = 15 + 15 \cdot 2(n - 1)$
- ☐ $w_n = 15 + 15 \cdot 2(n)$
- ☑ $w_n = 15 \cdot 2^{n-1}$
- ☐ $w_n = \frac{1}{2} \cdot 15 \cdot 2^{n-1}$
- ☐ $w_n = \frac{1}{2} \cdot 15 \cdot 2^{n}$
- ☑ $n \geq 1$, where n is an integer
- ☐ $n \geq 1$, where n is a real number
- ☐ n can be any real number

Week	Number of cells in medium
1	15
2	30
3	60
4	120
5	240
6	480

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10.) A local lottery offers its winners two prize options. Option A gives the lottery winner $100 on the Friday after they win. They will receive 1% more each Friday for 30 years after that. Option B gives the lottery winner $1000 on the Friday after they win and an additional $250 each week for 30 years.

A.) Write an explicit formula for each option.

Option A:

$A_N = 100(1.01)^{N-1}$

Option B:

$A_N = 1000 + (N-1)(250)$

$A_N = 1000 + 250N - 250$

$A_N = 250N + 750$

B.) How much money will the winner receive in the 15th year?

Option A:

$A_{780} = 100(1.01)^{780-1}$

$A_{780} = 232461.04$

Option B:

$A_{780} = 250(780) + 750$

$A_{780} = 195750$

C.) How much money will the winner have total after 30 years?

Option A:

$S_N = \frac{A_1(1-r^N)}{(1-r)}$

$S_{1560} = \frac{100(1-1.01^{1560})}{1-1.01}$

$S_{1560} = \$5.512 \times 10^{10}$

Option B:

$S_N = \frac{N}{2}(A_1 + A_N)$

$S_{1560} = \frac{1560}{2}(1000 + 390750)$

$S_{1560} = \$305565,000$

$A_{1560} = 1000 + (1560-1)(250)$

$A_{1560} = 390750$

D.) Which option is better for the lottery winner? Explain your answer.

OPTION A - MAKE MORE MONEY IN THE LONG RUN

11.) A child building a tower with blocks uses 15 for the bottom row. Each row has 2 fewer blocks than the previous row. There are 8 rows in the tower. How many blocks make up the tower?

15, 13, 11, ...

$S_N = \frac{N}{2}(A_1 + A_N)$

$S_8 = \frac{8}{2}(15 + 1)$

$S_8 = 4(16)$

$S_8 = 64$

$A_8 = 15 + (8-1)(-2)$

$A_8 = 15 + (7)(-2)$

$A_8 = 1$

$\boxed{\text{64 BLOCKS}}$

12.) There is a stack of logs in the backyard. There are 15 logs in the 1st layer, 14 in the second, 13 in the third, 12 in the fourth, and so on with the last layer having one log. How many logs are in the stack?

$S_N = \frac{N}{2}(A_1 + A_N)$

$S_{15} = \frac{15}{2}(15 + 1)$

$S_{15} = \frac{15}{2}(16)$

$S_{15} = 120$

$A_N = A_1 + (N-1)(D)$

$1 = 15 + (N-1)(-1)$

$1 = 15 - N + 1$

$1 = 16 - N$

$-15 = -N$

$15 = N$

$\boxed{\text{120 LOGS}}$

13.) In his piggy bank, Bingo dropped \$1.00 on May 1, \$1.75 on May 2, \$2.50 on May 3 and so on until the last day of May.

$1.75,\ 2.50,\ 3.25, \ldots$

A.) How much did he drop in his piggy bank on May 19?

$A_N = A_1 + (N-1)(D)$

$A_{19} = 1.75 + (19-1)(.75)$

$A_{19} = 1.75 + (18)(.75)$

$A_{19} = 15.25$

$\boxed{\$15.25}$

B.) What was his total deposit in his piggy bank for the month of May?

$S_N = \frac{N}{2}(A_1 + A_N)$

$S_{31} = \frac{31}{2}(1.75 + 24.25)$

$S_{31} = 403$

$A_N = A_1 + (N-1)(D)$

$A_{31} = 1.75 + (31-1)(.75)$

$A_{31} = 1.75 + (30)(.75)$

$A_{31} = 24.25$

$\boxed{\$403}$

14.) There are 20 rows of seats on a concert hall: 25 seats are in the 1st row, 27 seats on the 2nd row, 29 seats on the 3rd row, and so on. If the price per ticket is \$2,300, how much will be the total sales for a one-night concert if all seats are taken?

$S_N = \frac{N}{2}(A_1 + A_N)$

$S_{20} = \frac{20}{2}(25 + 63)$

$S_{20} = 10(88)$

$S_{20} = 880$

$A_N = A_1 + (N-1)(D)$

$A_{20} = 25 + (20-1)(2)$

$A_{20} = 25 + (19)(2)$

$A_{20} = 63$

TOTAL SEATS: 880

880 × \$2300

TOTAL SALES: \$2024,000

15.) A child building a tower with blocks uses 15 for the bottom row. Each row has 2 fewer blocks than the previous row. Suppose that there are 8 rows in the tower.

$15, 13, 11$

A.) How many blocks are used for the top row?

$A_N = A_1 + (N-1)(D)$

$A_8 = 15 + (8-1)(-2)$

$A_8 = 15 + (7)(-2)$

$A_8 = 1$

1 BLOCK IN TOP ROW

B.) What is the total number of blocks in the tower?

$S_N = \frac{N}{2}(A_1 + A_N)$

$S_8 = \frac{8}{2}(15 + 1)$

$S_8 = 64$

64 TOTAL BLOCKS

16.) A rider mower is purchased in 2010 for $2500. Each year, the value of the mower decreases by 15%...

A.) Fill in the table below to show the value of the rider mower in the first 5 years.

year	Balance
1	$2,500
2	2125
3	1806.25
4	1535.31
5	1305.01

B.) Write an explicit formula for the value of the rider mower in "n" years.

$$A_N = 2500\,(.85)^{N-1}$$

C.) What will be the value of the mower in 10 years?

$$A_{10} = 2500\,(.85)^{10-1}$$

$$A_{10} = 2500\,(.85)^{9}$$

$$A_{10} = 579.04$$

$\boxed{\$579.04}$

D.) How long will it take for the mower to be worth $1000?

$$\frac{1000}{2500} = \frac{2500\,(.85)^{N-1}}{2500}$$

$$.4 = (.85)^{N-1}$$

$$\log_{.85} .4 = N-1$$

$$N = (\log_{.85} .4) + 1$$

$$N = 6.6381$$

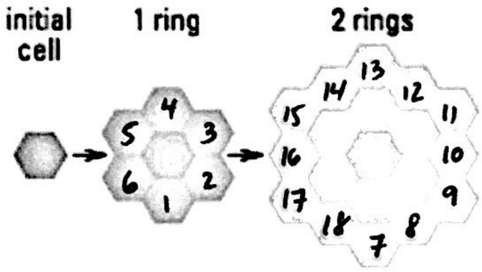
$\boxed{6.6381 \text{ YEARS}}$

17.) Domestic bees make their honeycomb by starting with a single hexagonal cell, then forming ring after ring of hexagonal cells around the initial cell, as shown below.

A.) Fill in the table.

IN THAT RING

Rings	# of Cells
0	1
1	6
2	12
3	18
4	24
5	30



B.) Write a rule for the number of cells in the nth ring.

$$A_N = A_1 + (N-1)(D)$$

$$A_N = 6 + (N-1)(6)$$

$$A_N = 6 + 6N - 6$$

$$\boxed{A_N = 6N}$$

C.) What is the total number of cells in the honeycomb after the 9th ring is formed?

$$S_N = \frac{N}{2}(A_1 + A_N)$$

$$S_9 = \frac{9}{2}(6 + 54)$$

$$S_9 = 270$$

$$A_N = 6N$$

$$A_9 = 6 \cdot 9$$

$$A_9 = 54$$

TOTAL CELLS:

270 + 1

THE INITIAL CELL

$\boxed{271 \text{ CELLS}}$

RETRO QUESTIONS:

Graph. Fill in important information.

18.) $y = (2)(3)^{x+4} - 5$

$y = (2)(3)^x$

x	y
0	2
1	6

Growth or Decay: GROWTH

Parent Function: $y = 3^x$

Asymptote: $y = -5$

Domain: $(-\infty, \infty)$

Range: $(-5, \infty)$

End Behavior:

as $x \to \infty, y \to$ ∞

as $x \to \infty, y \to$ -5

y–intercept: $(0, 157)$

$y = (2)(3)^{0+4} - 5$

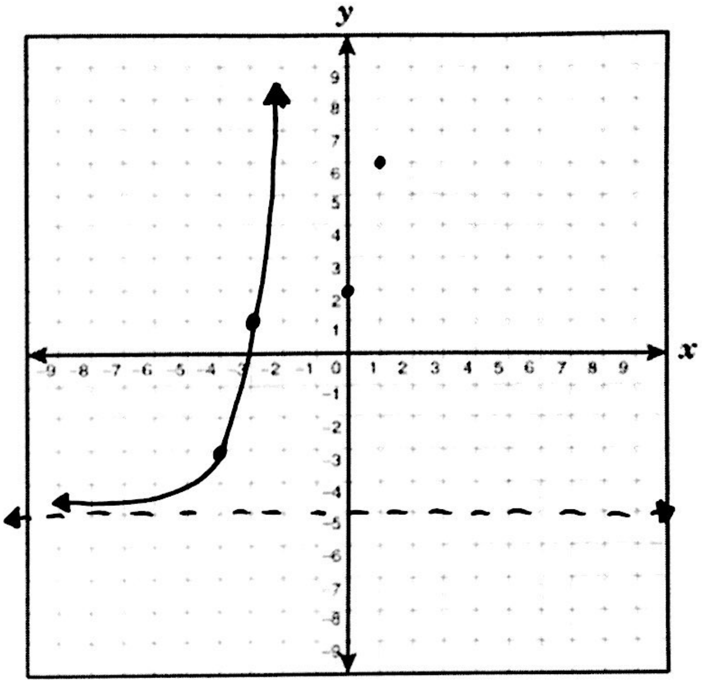
$y = (2)(3)^4 - 5$

$y = (2)(81) - 5$

$y = 157$

Transformation(s): VERTICLE STRETCH OF 2

LEFT 4, DOWN 5



Solve.

19.) $2^{x-3} = 16$

$2^{x-3} = 2^4$

$x - 3 = 4$

$x = 7$

20.) $\log(2x - 6) = 2$

$10^2 = 2x - 6$

$100 = 2x - 6$

$106 = 2x$

$x = 53$

21.) $e^{3x-1} = 4$

$\ln 4 = 3x - 1$

$(\ln 4) + 1 = 3x$

$x = \frac{(\ln 4) + 1}{3}$

$x = 0.7954$

Simplify.

22.) $\frac{x+4}{x^2-4x-5} + \frac{x+3}{x+1} \cdot \frac{(x-5)}{(x-5)}$

$(x-5)(x+1)$

$= \frac{x+4}{(x-5)(x+1)} + \frac{x^2-2x-15}{(x-5)(x+1)}$

$= \frac{x^2-x-11}{(x-5)(x+1)}$

23.) $\frac{x^2-16}{x^2-6x+5} \div \frac{2x+8}{x^2-3x+2}$

$= \frac{x^2-16}{x^2-6x+5} \cdot \frac{x^2-3x+2}{2x+8}$

$= \frac{(x-4)\cancel{(x+4)}}{(x-5)\cancel{(x-1)}} \cdot \frac{(x-2)\cancel{(x-1)}}{2\cancel{(x+4)}}$

$= \frac{(x-4)(x-2)}{2(x-5)}$