

## Angles

**RAY:** a line segment that continues in one direction.

**ANGLE:** a ray rotating around its endpoints; can move clockwise or counterclockwise.

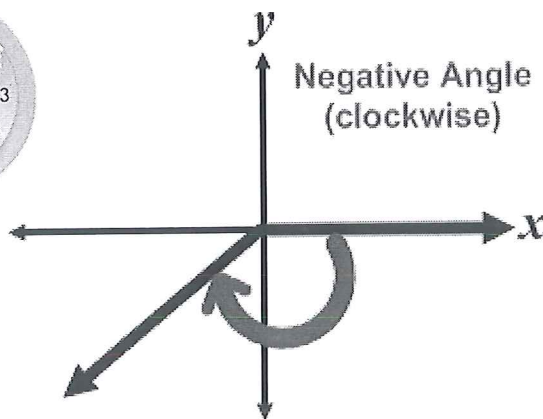
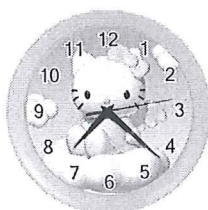
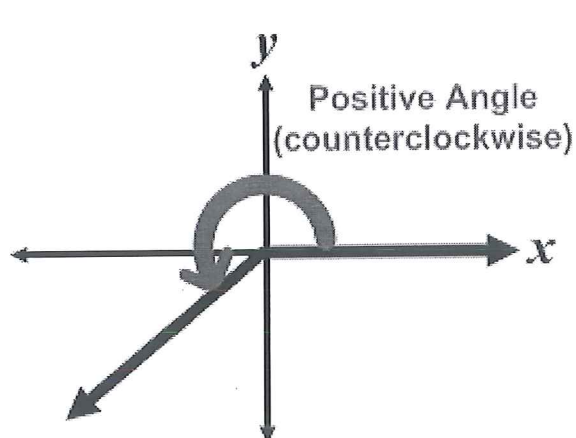
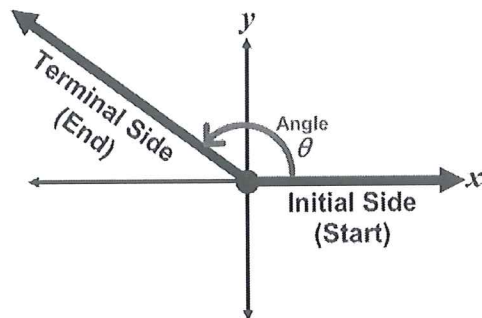
**DEGREE:** most common unit for measuring angles.

**INITIAL SIDE:** the beginning side of an angle.

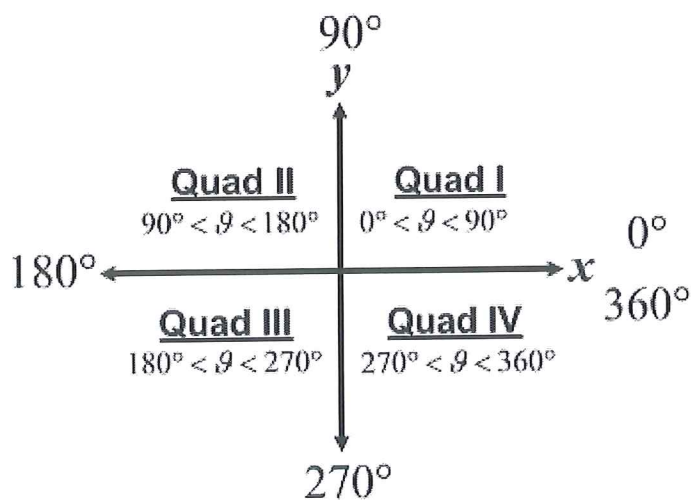
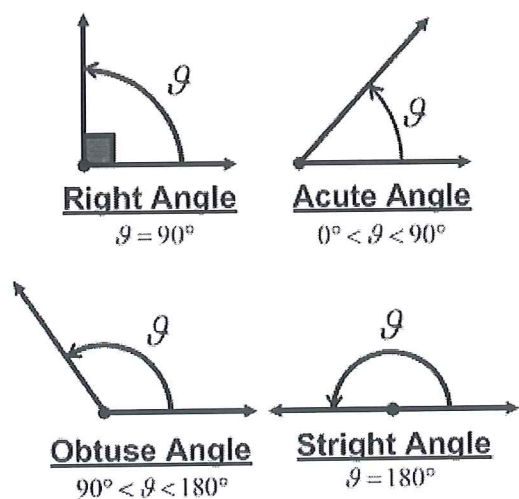
**TERMINAL SIDE:** the ending side of an angle.

**POSITIVE ANGLE:** angle formed when moved counterclockwise from the initial side.

**NEGATIVE ANGLE:** angle formed when moved clockwise from the initial side.



### Types of Angles and Coordinate Plane

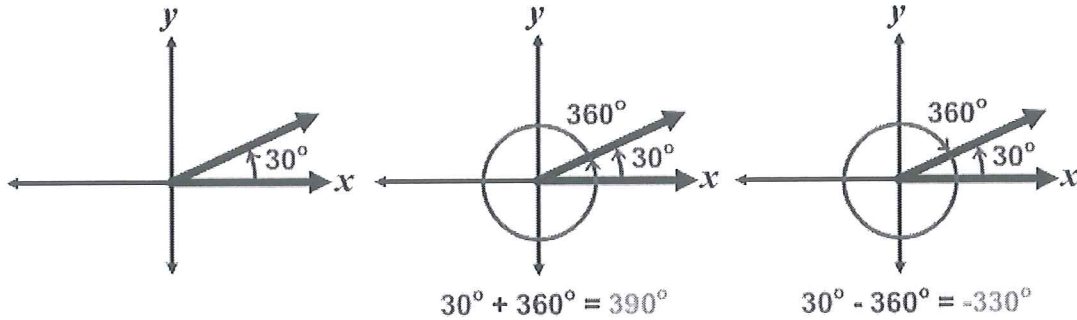


# Coterminal Angles

**COTERMINAL ANGLES:** two angles that have the same initial and terminal sides.

To find a coterminal angle to a given angle, add or subtract  $360^\circ$ .

To find angles of *smallest positive measure* to a given angle, add or subtract multiples of  $360^\circ$ . Angles must be positive and between  $0^\circ$  and  $360^\circ$ .



1. Find two coterminal angles to  $167^\circ$ .

$$167^\circ + 360^\circ = 527^\circ$$

$$167^\circ - 360^\circ = -193^\circ$$

2. Find the angle of *smallest positive measure* coterminal with each given angle.

(a)  $-135^\circ$

$$-135^\circ + 360^\circ = 225^\circ$$

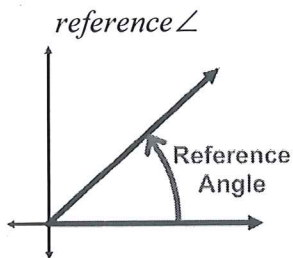
(b)  $1200^\circ$

$$1200^\circ - 360^\circ - 360^\circ - 360^\circ = 120^\circ$$

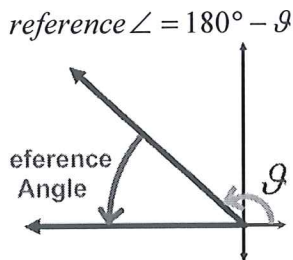
## Reference Angles

**REFERENCE ANGLE:** a positive acute angle made by the terminal side of an angle and the  $x$ -axis.

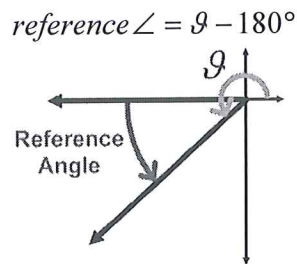
Quadrant I



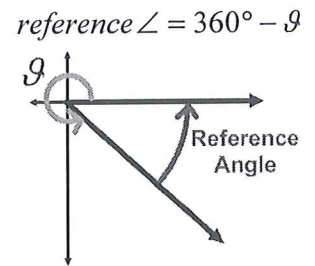
Quadrant II



Quadrant III



Quadrant IV



**State the quadrant of each given angle and then find its reference angle.**

3.  $165^\circ$

Quadrant II

Ref.  $\angle =$   $15^\circ$

$$180^\circ - 165^\circ = 15^\circ$$

4.  $254^\circ$

Quadrant III

Ref.  $\angle =$   $16^\circ$

$$270^\circ - 254^\circ = 16^\circ$$

5.  $-200^\circ$

Quadrant II

Ref.  $\angle =$   $20^\circ$

$$180^\circ - 160^\circ = 20^\circ$$

6.  $1387^\circ$

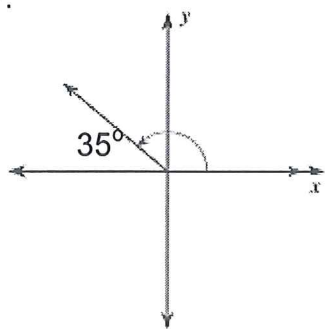
Quadrant IV

Ref.  $\angle =$   $53^\circ$

$$360^\circ - 307^\circ = 53^\circ$$

Find the measure of each angle that is in standard position.

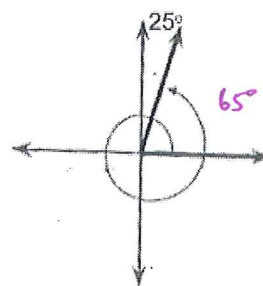
7.



$$180^\circ - 35^\circ$$

$$\boxed{145^\circ}$$

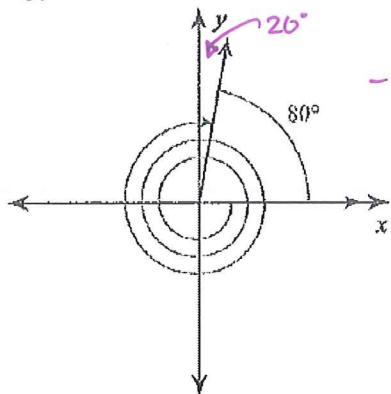
8.



$$360^\circ + 65^\circ$$

$$\boxed{425^\circ}$$

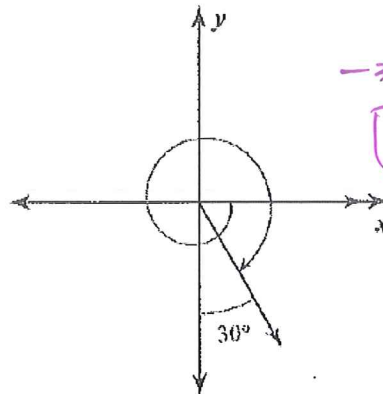
9.



$$-360^\circ - 360^\circ - 270^\circ - 20^\circ$$

$$\boxed{-1010^\circ}$$

10.

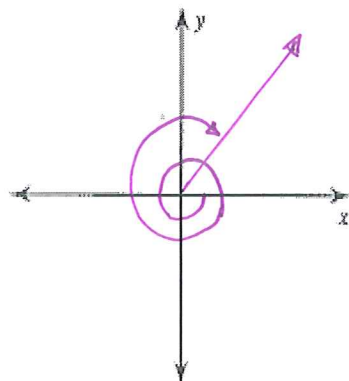


$$-360^\circ - 70^\circ$$

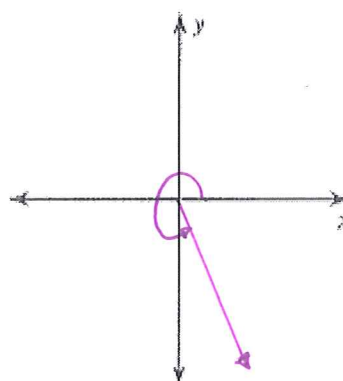
$$\boxed{-430^\circ}$$

Draw an angle with the given measure in standard position. Then state its quadrant and find its reference angle.

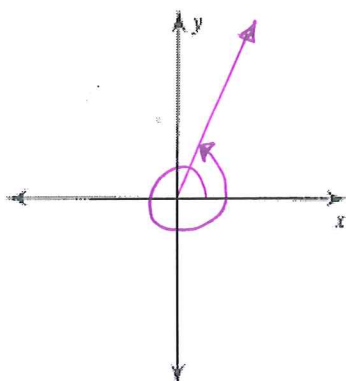
11.  $-670^\circ$  Quadrant I Ref.  $\angle =$   $50^\circ$



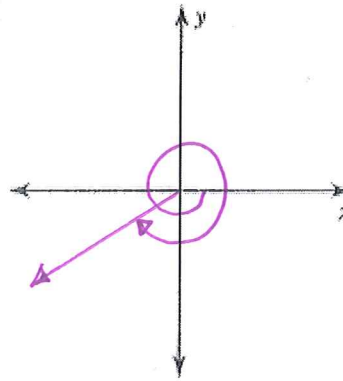
12.  $275^\circ$  Quadrant IV Ref.  $\angle =$   $85^\circ$



13.  $440^\circ$  Quadrant I Ref.  $\angle =$   $80^\circ$



14.  $-509^\circ$  Quadrant III Ref.  $\angle =$   $31^\circ$



State the quadrant in which the terminal side of each angle lies and then find the reference angle.

13.  $-189^\circ$

$-189^\circ + 360^\circ = 171^\circ$

Quadrant II

$180^\circ - 171^\circ = 9^\circ$

Ref.  $\angle =$   $9^\circ$

14.  $920^\circ$

$920^\circ - 360^\circ - 360^\circ = 200^\circ$

Quadrant III

$200^\circ - 180^\circ = 20^\circ$

Ref.  $\angle =$   $20^\circ$

15.  $-310^\circ$

$-310^\circ + 360^\circ = 50^\circ$

Quadrant I

Ref.  $\angle =$   $50^\circ$

16.  $1430^\circ$

$1430^\circ - 360^\circ - 360^\circ - 360^\circ = 350^\circ$

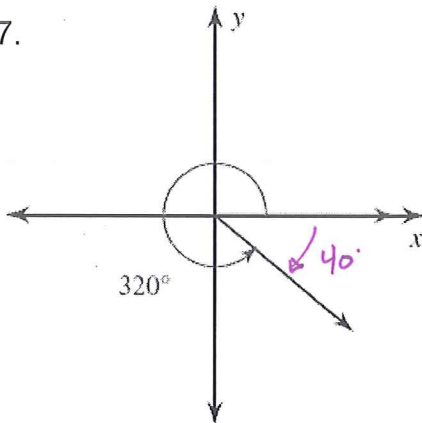
Quadrant IV

$360^\circ - 350^\circ = 10^\circ$

Ref.  $\angle =$   $10^\circ$

Find the reference angle.

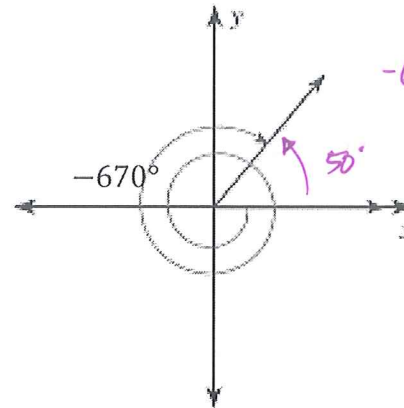
17.



$360^\circ - 320^\circ$

$40^\circ$

18.

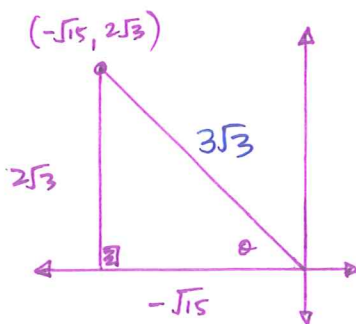


$-670^\circ + 360^\circ + 360^\circ$

$50^\circ$

18. Find the values of the six, simplified trigonometric functions of angle  $\theta$ .

Point  $(-\sqrt{15}, 2\sqrt{3})$ .



$(-\sqrt{15})^2 + (2\sqrt{3})^2 = r^2$

$15 + 12 = r^2$

$27 = r^2$

$\sqrt{27} = r$

$\sqrt{9 \cdot 3} = r$

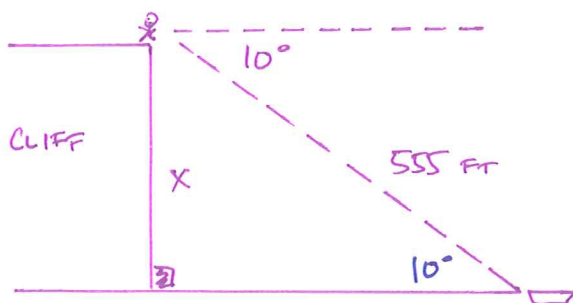
$3\sqrt{3} = r$

$\sin \theta = \frac{2\sqrt{3}}{3\sqrt{3}} = \frac{2}{3}$        $\csc \theta = \frac{3}{2}$

$\cos \theta = \frac{-\sqrt{15}}{3\sqrt{3}}$        $\sec \theta = \frac{3\sqrt{3}}{-\sqrt{15}}$

$\tan \theta = \frac{2\sqrt{3}}{-\sqrt{15}}$        $\cot \theta = \frac{-\sqrt{15}}{2\sqrt{3}}$

19. A person at the edge of a cliff looks down at a boat on a lake. The angle of depression of a person's line of sight is  $10^\circ$  and the line of sight distance from the person to the boat is 555 feet. How high is the cliff?



$\sin 10^\circ = \frac{X}{555}$

$X = 555 \sin 10^\circ$

$X = 96.3747 \text{ ft}$